

Bound-state versus collective-coordinate approaches in chiral soliton models and the width of the Θ^+ pentaquark

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Abstract. We thoroughly compare the bound-state and rigid-rotator approaches to three-flavored chiral solitons. We establish that these two approaches yield identical results for the baryon spectrum and kaon-nucleon S -matrix in the limit that the number of colors (N_C) tends to infinity. After proper subtraction of the background phase shift the bound-state approach indeed exhibits a clear resonance behavior in the strangeness $S = +1$ channel. We present a first dynamical calculation of the widths of the Θ^+ and Θ^* pentaquarks for finite N_C in a chiral soliton model.

PACS. 12.39.Dc Skyrmions – 13.30.Eg Hadronic decays – 13.75.Jz Kaon-baryon interactions

1 Introduction

Chiral soliton model predictions for the mass of the lightest exotic pentaquark, the Θ^+ with zero isospin and unit strangeness, have been around for about two decades [1–6]. Nevertheless, the study of such pentaquarks as potential baryon resonances became popular only a short while ago when experiments [7] indicated their existence¹. These experiments were stimulated by a chiral soliton model estimate that claimed that the width of the pentaquark decaying into a kaon and a nucleon would be much smaller than those typical for hadronic decays of baryon resonances [10–12]; however, see also [13]. Such narrow resonances could have escaped detection in earlier analyses.

1.1 Hadronic decays in soliton models

Estimates for the Θ^+ width are based on identifying the interpolating kaon field with the (divergence) of the axial current, which, in turn, is computed solely from the classical soliton. This is essentially a generalization to flavor $SU(3)$ of the early calculation of the $\Delta \rightarrow \pi N$ decay width in the pioneering paper of ref. [14] in the two-flavor Skyrme model. Since then there have been quite

some efforts to understand that process in chiral soliton models by going beyond the simple identification of the pseudoscalar meson fields with the axial current [15–23]; see also chapt. IV in ref. [24]. It must be stressed that the description of hadronic decays in soliton models is a severe problem and has become known as the *Yukawa* problem. The reason is that the interaction Hamiltonian that mediates such hadronic transitions must be linear in the field operator that corresponds to the pseudoscalar meson in the final state. On the other hand the naïve definition of the soliton as a stationary point of the action prohibits such a linear term. Hence, in these models, resonance widths must be extracted from appropriate S -matrix elements of meson baryon scattering. Most of the studies on meson baryon scattering are restricted to the adiabatic approximation [25–28] which represents the next-to-leading order in the $1/N_C$ expansion, where N_C is the number of colors. Unfortunately, in the large- N_C limit the Δ is degenerate with the nucleon and its width is of even lower order in that expansion. Therefore its width cannot be computed in adiabatic approximation and the above-cited studies mainly motivate Born contributions to the S -matrix element of pion nucleon scattering in the P_{33} channel at $\mathcal{O}(1/N_C)$. For the pentaquarks the situation is very different. As we will see, the Θ^+ -nucleon mass difference is non-zero even as $N_C \rightarrow \infty$ and hence its hadronic-decay width should be at the same order as the adiabatic approximation for kaon nucleon scattering. In that respect the hadronic decays of pentaquarks are a perfect playground: Any treatment that induces Born contributions can be tested against the adiabatic approximation.

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¹ For recent reviews on the experimental situation we refer to refs. [8,9]. The numerous theory papers that have appeared since then may be downloaded from <http://www.rcnp.osaka-u.ac.jp/~hyodo/research/Thetapub.html>.

1.2 Quantizing the soliton in flavor $SU(3)$

When treating chiral solitons in flavor $SU(3)$ we face the problem of how to properly generate baryon states from the classical soliton. Essentially there are two methods at our disposal, the bound-state approach (BSA) [29] and the rigid-rotator approach (RRA) [30]; see, *e.g.*, ref. [31] for a review and further references. The latter is essentially the generalization of the collective-coordinate quantization adopted earlier for the two-flavor model [14]. In this approach collective coordinates that describe the spin flavor orientation of the soliton are introduced and quantized canonically. As this corresponds to quantizing a rigid top in $SU(3)$ it is natural that the corresponding eigenstates are characterized according to $SU(3)$ multiplets, most prominently the octet (**8**) and decuplet (**10**) for baryons with spin $J = \frac{1}{2}$ and $J = \frac{3}{2}$, respectively. However, also more exotic $SU(3)$ representations like the antidecuplet (**$\bar{10}$**) emerge. In addition to states with quantum numbers of octet baryons, this representation also contains states with quantum numbers that *cannot* be built as three-quark composites. These states contain (at least) one additional quark-antiquark pair which lead to the notion of exotic pentaquarks. Most prominently there have been experimental claims [7–9] for a Θ^+ pentaquark, the strangeness $S = +1$ isosinglet that sits at the top of the antidecuplet.

In the BSA, kaon fluctuations $\eta_\alpha(\vec{x}, t)$, with $\alpha = 4, \dots, 7$, are introduced and quantized as harmonic oscillations. This corresponds to the adiabatic approximation and should be exact as $N_C \rightarrow \infty$. Common to all soliton models is the appearance of a bound state with strangeness $S = -1$ which serves to construct states that correspond to the ordinary hyperons, Λ , Σ and Σ^* . On the other hand, neither a bound state nor a clear resonance has been observed in the $S = +1$ channel. This non-observation has often been used to argue that the pentaquark prediction would be a mere artifact of the RRA [32–34]. Here we will show that this statement is completely based on a misinterpretation of the RRA. For a sensible and ultimate comparison of the BSA and the RRA it is also necessary to include small amplitude fluctuations in the RRA. We call that approach the rotation-vibration approach (RVA). However, the fluctuations in the RVA must be constrained to the subspace that is orthogonal to the rigid rotations. We therefore label these fluctuations by $\bar{\eta}_\alpha(\vec{x}, t)$. Then the test mentioned at the end of subsect. 1.1 is to first compute the S -matrix from the constrained fluctuations, $\bar{\eta}_\alpha(\vec{x}, t)$ and to supplement that result with the contributions from the (iterated) Θ^+ exchange. That yields the wave functions $\tilde{\eta}_\alpha(\vec{x}, t)$ that are constrained to the same subspace as $\bar{\eta}_\alpha(\vec{x}, t)$. The $\tilde{\eta}_\alpha(\vec{x}, t)$ are the full solution to the RVA. In the limit $N_C \rightarrow \infty$ $\tilde{\eta}_\alpha(\vec{x}, t)$ must yield the same S -matrix as in the BSA, *i.e.* when extracted from $\eta_\alpha(\vec{x}, t)$. In performing this test, our studies serve three major purposes: First, we will establish the equivalence of the BSA and the RVA in the large- N_C limit. Second, we will compare $\tilde{\eta}_\alpha$ and $\bar{\eta}_\alpha$ to extract a distinct $S = 1$ resonance in kaon nucleon scattering that survives as $N_C \rightarrow \infty$. Third, we will provide

the *first* dynamical calculation of the Θ^+ width in a chiral soliton model for finite N_C .

1.3 General remarks

Let us round off this introduction section with a few general remarks on the comparison between the BSA, the RRA and the RVA. Generally we may always introduce collective coordinates to investigate specific excitations. This is irrespective of whether the corresponding excitation energies are suppressed in the large- N_C limit or not. That is, in contrast to the claims of refs. [33] no separation of scales is required to validate the collective-coordinate quantization. Eventually the coupling terms and constraints ensure correct results. Moreover, we will observe in sect. 5 that collective coordinates may also be introduced for $m_K \neq m_\pi$. This result generalizes to the statement that collective coordinates are *not* necessarily linked to the zero-modes of a system and any distinction between dynamical and static zero-modes [34] is obsolete. Already some time ago the concept of describing baryon resonances as collective excitations has been successfully applied to quadrupole [35] and monopole excitations, both in two [36] and three [37] flavor models. In particular collective monopole excitations have been studied in the context of the pentaquark excitation [38, 39].

Sometimes the correct description of the $\Lambda(1405)$ as an S -wave bound state is considered as an argument in favor of the BSA *vs.* the RRA. Again it must be stressed that the full RVA also contains kaon fluctuations and thus also describes the formation of an S -wave bound state. As shown in ref. [40] this properly accounts for the $\Lambda(1405)$ in the RVA.

1.4 Outline

This paper is organized as follows. In sect. 2 we will define the model and describe the field parameterization in the bound-state and rigid-rotator approaches in more detail. In sects. 3 and 4 we extensively compare these approaches and show how they yield identical spectra in the limit $N_C \rightarrow \infty$, even for non-zero symmetry breaking, when the fluctuating modes are limited to the same subspace. In sect. 3 we also show that the constraints for $\bar{\eta}_\alpha$ serve to separate resonance and background pieces of the phase shift. Section 5 represents a main piece of our paper as we combine rotational and fluctuation degrees of freedom to construct the solution for the RVA up to quadratic order in the fluctuations. We employ this solution to compute the width of the Θ^+ pentaquark for $m_K = m_\pi$ in sect. 6. Another major result is that in the flavor symmetric case only a *single* collective-coordinate operator mediates the transition $\Theta^+ \rightarrow KN$ at the leading order of the large- N_C expansion. This is in direct contradiction to approaches that construct transition interactions with (at least) two $SU(3)$ operators and adjust the coefficients to yield a narrow Θ^+ [10, 11, 41, 42]. In sect. 7 we extend our treatment to the physical case $m_K \neq m_\pi$. This brings a second $SU(3)$ operator into the game. In sect. 8 we collect our numerical results and summarize in sect. 9. We relegate technical

details on the derivation of the equation of motion for the constrained fluctuations to an appendix.

2 The model

For simplicity we consider the Skyrme model [43] as a particular example for chiral soliton models. However, we stress that our qualitative results do indeed generalize to *all* chiral soliton models because these results solely originate from the treatment of the model degrees of freedom.

A preliminary remark on notation: In what follows we adopt the convention that repeated indices are summed over in the range

$$\begin{aligned} a, b, c, \dots &= 1, \dots, 8, \\ \alpha, \beta, \gamma, \dots &= 4, \dots, 7 && \text{(kaonic)}, \\ i, j, k, \dots &= 1, 2, 3 && \text{(pionic)}. \end{aligned} \quad (2.1)$$

Chiral soliton models are functionals of the chiral field, U , the non-linear realization of the pseudoscalar mesons, ϕ_a ,

$$U(\vec{x}, t) = \exp \left[\frac{i}{f_\pi} \phi_a(\vec{x}, t) \lambda_a \right], \quad (2.2)$$

with λ_a being the Gell-Mann matrices of $SU(3)$. For a convenient presentation of the model we split the action into three pieces,

$$\Gamma = \Gamma_{\text{SK}} + \Gamma_{\text{WZ}} + \Gamma_{\text{SB}}. \quad (2.3)$$

The first term represents the Skyrme model action,

$$\begin{aligned} \Gamma_{\text{SK}} = \int d^4x \operatorname{tr} \left\{ \frac{f_\pi^2}{4} [\partial_\mu U \partial^\mu U^\dagger] \right. \\ \left. + \frac{1}{32\epsilon^2} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \right\}. \end{aligned} \quad (2.4)$$

Here $f_\pi = 93 \text{ MeV}$ is the pion decay constant and ϵ is the dimensionless Skyrme parameter. In principle this is a free model parameter. The two-flavor version of the Skyrme model suggests to put $\epsilon = 4.25$ from reproducing the Δ -nucleon mass difference. The anomaly structure of QCD is incorporated via the Wess-Zumino action [44]

$$\Gamma_{\text{WZ}} = -\frac{iN_C}{240\pi^2} \int d^5x \epsilon^{\mu\nu\rho\sigma\tau} \operatorname{tr} [L_\mu L_\nu L_\rho L_\sigma L_\tau], \quad (2.5)$$

with $L_\mu = U^\dagger \partial_\mu U$. Note that Γ_{WZ} vanishes in the two-flavor version of the model. Furthermore, N_C is the number of colors, the hidden expansion parameter of QCD [45, 46].

The flavor symmetry breaking terms are contained in Γ_{SB} ,

$$\begin{aligned} \Gamma_{\text{SB}} = \frac{f_\pi^2}{4} \int d^4x \operatorname{tr} [\mathcal{M} (U + U^\dagger - 2)] \quad \text{with} \\ \mathcal{M} = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}. \end{aligned} \quad (2.6)$$

Unless otherwise stated, we will adopt the empirical data for the masses of the pseudoscalar mesons: $m_\pi = 138 \text{ MeV}$

and $m_K = 495 \text{ MeV}$. We do not include terms that distinguish between pion and kaon decay constants even though they differ by about 20% empirically. This is a mere matter of convenience rather than negligence. This omission leads to a considerable underestimation of symmetry breaking effects [47] which approximately can be accounted for by rescaling the kaon mass $m_K \rightarrow m_K f_K / f_\pi$.

Although physically sensible results are obtained by setting $N_C = 3$, it is very illuminating to organize the calculation in powers of $1/N_C$ and consider N_C undetermined. In the context of this N_C counting, the mass parameters are $\mathcal{O}(N_C^0)$ while $f_\pi = \mathcal{O}(\sqrt{N_C})$ and $\epsilon = \mathcal{O}(1/\sqrt{N_C})$ to ensure that the (perturbative) n -point functions scale as $N_C^{1-n/2}$ [46]. In order to study the N_C -dependence in the three-flavor version of the soliton model we include the N_C -dependence in the choice of the model parameters via

$$f_\pi = 93 \text{ MeV} \sqrt{\frac{N_C}{3}} \quad \text{and} \quad \epsilon = 4.25 \sqrt{\frac{3}{N_C}}. \quad (2.7)$$

The action, eq. (2.3) allows for a topologically non-trivial classical solution, the famous hedgehog soliton,

$$U_0(\vec{x}) = \exp \left[i \vec{\lambda} \cdot \hat{x} F(r) \right], \quad r = |\vec{x}|, \quad (2.8)$$

which is embedded in the isospin subspace of flavor $SU(3)$ parameterized by the Gell-Mann matrices λ_i , $i = 1, 2, 3$. Other embeddings are possible but energetically disfavored due to flavor symmetry breaking. The chiral angle, $F(r)$ is obtained from integrating the classical equation of motion extracted from eqs. (2.4) and (2.6) subject to the boundary condition² $F(0) - F(\infty) = \pi$ ensuring winding (baryon) number one. The soliton can be constructed as a function of the dimensionless variable $\epsilon f_\pi r$ and is thus not subject to N_C scaling.

The excitations of this soliton configuration are quantized to generate physical baryon states. The central task in extending this picture to three flavors is the incorporation of strange degrees of freedom, that is *kaons*. In a first step we introduce fluctuations³ $\eta_a(\vec{x}, t)$ with $a = 1, \dots, 8$, for the pseudoscalar fields [29, 48, 49],

$$U(\vec{x}, t) = \sqrt{U_0(\vec{x})} \exp \left[\frac{i}{f_\pi} \lambda_a \eta_a(\vec{x}, t) \right] \sqrt{U_0(\vec{x})}. \quad (2.9)$$

Expanding the action in powers of these fluctuations actually is an expansion in η_a/f_π and thus a systematic series in $1/\sqrt{N_C}$. The term quadratic in η_a is of special interest. It describes scattering of mesons off a potential that is generated by the classical soliton, eq. (2.8). In particular an antikaon is bound in the P -wave channel. Combined with the soliton it may be interpreted as the Λ -hyperon. However, the system consisting of the soliton and this bound antikaon has still to be projected onto states with

² The overall sign is a matter of convention.

³ In what follows we will discuss different parameterizations of the chiral field, U . We abstain from introducing additional labels to distinguish among them.

good spin and isospin. This is accomplished by introducing collective coordinates for the spin-isospin orientation⁴, $A_2(t) \in SU(2)$,

$$U(\vec{x}, t) = A_2(t) \sqrt{U_0(\vec{x})} \exp \left[\frac{i}{f_\pi} \lambda_a \eta_a(\vec{x}, t) \right] \sqrt{U_0(\vec{x})} A_2^\dagger(t) \quad (2.10)$$

and quantizing them canonically [29, 49]. In this treatment the kaon fluctuations $a = \alpha = 4, \dots, 7$ decouple in the adiabatic equations of motion. This leads to the so-called bound-state approach (BSA) that we will review in sect. 3.

The seemingly alternative approach is to allow the collective coordinates to span the *full* flavor space, $A_3(t) \in SU(3)$, and quantize A_3 canonically to generate baryon states with non-zero strangeness while omitting the fluctuations. Baryon states are then classified according to $SU(3)$ representations. This is the so-called rigid-rotator approach (RRA) [30]. However, for a sensible investigation of the rotation-vibration coupling, the collective-coordinate approach must be completed to also contain meson fluctuations. Hence, the *ansatz* that defines the RVA reads

$$U(\vec{x}, t) = A_3(t) \sqrt{U_0(\vec{x})} \exp \left[\frac{i}{f_\pi} \lambda_a \tilde{\eta}_a(\vec{x}, t) \right] \sqrt{U_0(\vec{x})} A_3(t)^\dagger. \quad (2.11)$$

Obviously eq. (2.11) comprises the *ansatz*, eq. (2.10). In contrast to eq. (2.10), the *ansatz* (2.11) has additional collective rotations into strange directions. But these field components are also contained in the fluctuations, $\tilde{\eta}_\alpha$. It is therefore necessary to constrain these fluctuations to the subspace that is orthogonal to the rigidly rotating hedgehog. These constraints modify the equations of motion for the fluctuations, such that $\tilde{\eta}_\alpha \neq \eta_\alpha$. This is not new: In principle we encounter the same problem already in $SU(2)$ where the *ansätze* eqs. (2.10) and (2.11) are identical. The difference to the $SU(3)$ case is that the redundant modes are automatically orthogonal to the rigid rotations when treating the $SU(2)$ model in adiabatic approximation. This changes, however, as soon as non-adiabatic contributions or external fields *e.g.* photons [50], are considered. We will discuss and derive the modifications that occur in the $SU(3)$ case in sect. 3 and a separate appendix, respectively.

It is obvious that the *ansätze* (2.10) and (2.11) span the same Fock space in the baryon number one sector. An essential difference, however, is that an expansion to a given order in η_a using eq. (2.10) contains the kaon fields only up to a fixed power in the $1/N_C$ expansion, while the *ansatz* (2.11) treats the collective rotations to *all* orders. Obviously the two treatments must yield identical results in the limit $N_C \rightarrow \infty$. This provides a thorough check on our calculations. We also stress that the RVA formulation induces terms in the action that are linear in $\tilde{\eta}_\alpha$ (in contrast the BSA does not give terms linear in η_α). This gives rise to Yukawa couplings from which we will evaluate widths of rotational excitations.

There have been earlier studies of the non-adiabatic rotation-vibration coupling in the Skyrme model. In the

$SU(2)$ version the contribution linear in the time derivative of A_2 has been shown to properly describe the Tomozawa-Weinberg relations for S -wave pion nucleon scattering [51]. Also in the two flavor model, the terms quadratic in the time derivative of the collective coordinates have been employed to incorporate thresholds and Δ -resonance exchange effects in pion nucleon scattering [20–22, 28]. In the three-flavor BSA the rotation-vibration coupling gives rise to the hyperfine mass splitting [49] that we will discuss in the following section. Also in $SU(3)$ meson baryon scattering has been investigated in the RVA [40]. Unfortunately, the strangeness, $S = +1$ channel, which contains the Θ^+ pentaquark, was not considered in that outstanding paper.

3 Bound-state approach (BSA)

In this section we will discuss the construction of P -wave baryons with strangeness $S = \pm 1$ in the framework of the BSA starting from the parameterization, eq. (2.9). Baryons with zero strangeness are treated as in two-flavor chiral soliton models [14].

3.1 Conventional BSA in the P-wave channel

As in refs. [29, 48] we expand the action to quadratic order in the kaonic fluctuations η_α in the background of the soliton, eq. (2.8). These wave functions describe kaon nucleon scattering in the intrinsic frame. In this frame the modes decouple with respect to the grand spin $\mathcal{G} = \ell \pm \frac{1}{2}$, the eigenvalue of the sum of kaon orbital angular momentum (ℓ) and isospin. This defines the intrinsic T -matrix, $T_{\mathcal{G}, \ell}$. The T -matrix elements in the laboratory frame $T(\ell I 2 J)$ are linear combinations of the $T_{\mathcal{G}, \ell}$ [26, 52]. For the Θ^+ channel ($I = 0, J = \frac{1}{2}$) this linear relation is very simple,

$$T(P01) = T_{\frac{1}{2}, 1}, \quad (3.1)$$

i.e. only the intrinsic P -wave kaon with grand spin $\mathcal{G} = \frac{1}{2}$ contributes. Its wave function is characterized by a single radial function, that parametrically depends on the frequency in a Fourier analysis,

$$\begin{pmatrix} \eta_4 + i\eta_5 \\ \eta_6 + i\eta_7 \end{pmatrix}_P(\vec{x}, t) = \int_{-\infty}^{\infty} d\omega e^{i\omega t} \eta_\omega(r) \hat{x} \cdot \vec{\tau} \chi(\omega). \quad (3.2)$$

Here $\chi(\omega)$ is a two-component isospinor that contains the amplitudes of the kaon mode. Upon quantization its components will eventually be elevated to creation and annihilation operators. Of course, an analogous decomposition exists for $\tilde{\eta}_\alpha$ that defines the radial function $\tilde{\eta}_\omega(r)$. In what follows we will omit the subscript ω . The amplitude χ drops out from the equation of motion that can be formulated for the radial functions,

$$\begin{aligned} \Pi(r) &= [\omega M_K(r) - \lambda(r)] \eta(r), \\ h^2 \eta(r) + \frac{\lambda^2(r)}{M_K(r)} &= \left[\omega - \frac{\lambda(r)}{M_K(r)} \right] \Pi(r). \end{aligned} \quad (3.3)$$

⁴ Due to the hedgehog symmetry only one set of collective coordinates is required.

Up to a factor i the radial function $\Pi(r)$ parameterizes the momentum conjugate to the P -wave fluctuation [53]. The radial functions in eq. (3.3) can be taken from the literature [29, 48, 54] in terms of the chiral angle $F = F(r)$,

$$\begin{aligned}\lambda(r) &= \frac{N_C}{4f_\pi^2} B_0(r) = -\frac{N_C}{8\pi^2 f_\pi^2} F' \frac{\sin^2 F}{r^2}, \\ M_K(r) &= 1 + \frac{1}{4f_\pi^2 \epsilon^2} \left[F'^2 + 2 \frac{\sin^2 F}{r^2} \right] \\ h^2 &= h_0^2 + m_K^2 - m_\pi^2 \quad \text{with} \\ h_0^2 &= -\frac{d^2}{dr^2} - \frac{2}{r} \frac{d}{dr} + V_{\text{eff}}(r),\end{aligned}\quad (3.4)$$

where $F' = \frac{dF(r)}{dr}$. The effective potential $V_{\text{eff}}(r)$ is quite an involved function that also contains spatial derivative operators. Since it does not play any special role for our studies, we do not repeat it here. The radial function $\lambda(r)$, proportional to the baryon density, $B_0(r)$, originates from the WZ-term, eq. (2.5) and thus contains the explicit factor N_C . The radial function $M_K(r)$ is the metric function for kaon fluctuations. This becomes obvious when eliminating the momentum $\Pi(r)$ in favor of a second-order differential equation for $\eta(r)$,

$$h^2 \eta(r) + \omega [2\lambda(r) - \omega M_K(r)] \eta(r) = 0, \quad (3.5)$$

as $M_K(r)$ multiplies the term of highest power in the frequency, ω . Equation (3.5) could have been easily obtained without reference to the conjugate momentum. However, we have introduced it here because it becomes essential when implementing the constraints that characterize $\bar{\eta}$. The appearance of factors N_C/f_π^2 and $f_\pi^2 \epsilon^2$ in the radial functions shows that $\eta(r)$ does not scale with N_C . Hence, terms in the action that are of higher than quadratic order in η_α are suppressed due to the additional factor $1/f_\pi$. Thus, solving eq. (3.5) provides the *exact* T -matrix in the limit $N_C \rightarrow \infty$.

The equation of motion (3.5) is not invariant under particle conjugation $\omega \leftrightarrow -\omega$, yielding different results for kaons ($\omega > 0$) and antikaons ($\omega < 0$). This difference obviously originates from the Wess-Zumino term. It is well known that eq. (3.5) has a bound-state solution at $\omega = -\omega_\Lambda$, that gives the mass difference between the Λ -hyperon and the nucleon in the large- N_C limit. As this energy eigenvalue is negative, it corresponds to a kaon, *i.e.* it carries strangeness $S = -1$. In the symmetric case ($m_K = m_\pi$) the bound-state energy vanishes, *i.e.* the bound state is nothing but the zero-mode of $SU(3)$ flavor symmetry. The radial function of the properly normalized zero-mode reads

$$\begin{aligned}z(r) &= \sqrt{4\pi} \frac{f_\pi}{\sqrt{\Theta_K}} \sin \frac{F(r)}{2} \quad \text{with} \\ \Theta_K &= f_\pi^2 \int d^3r M_K(r) \sin^2 \frac{F(r)}{2} = 1.894 \text{ GeV}^{-1} \left(\frac{N_C}{3} \right).\end{aligned}\quad (3.6)$$

It satisfies the important relation $h_0^2 z(r) = 0$. The normalization constant, Θ_K , is the moment of inertia for flavor

rotations into strangeness direction. In what follows we will employ the expressions ‘‘zero-mode’’ and ‘‘rotational mode’’ synonymously.

When symmetry breaking is switched on, the bound state evolves continuously from this zero-mode. For $m_K = 495$ MeV we find $\omega_\Lambda = 196$ MeV. This bound-state energy can be reliably estimated by sandwiching the equation of motion (3.5) between zero-mode wave functions and solving for ω . We first define

$$\omega_0 = \int r^2 dr z^2(r) 2\lambda(r) = \frac{N_C}{4\Theta_K} = 0.396 \text{ GeV} \quad (3.7)$$

and

$$\begin{aligned}\Gamma &= \frac{8\Theta_K}{3} (m_K^2 - m_\pi^2) \int r^2 dr z^2(r) = \\ &2.969 \text{ GeV}^{-1} (m_K^2 - m_\pi^2) \frac{N_C}{3},\end{aligned}\quad (3.8)$$

which are $\mathcal{O}(N_C^0)$ and $\mathcal{O}(N_C)$, respectively. Then the equation for ω reads

$$\omega^2 = \frac{3\Gamma}{8\Theta_K} + \omega_0 \omega. \quad (3.9)$$

This quadratic equation for ω has the negative valued solution $-\omega_\Lambda$ with

$$\omega_\Lambda = \frac{1}{2} \left[\sqrt{\omega_0^2 + \frac{3\Gamma}{2\Theta_K}} - \omega_0 \right]. \quad (3.10)$$

For $m_K = m_\pi$ we have $\Gamma = 0$ and thus also $\omega_\Lambda = 0$ from this estimate. In the realistic case we find $\Gamma = 671$ MeV $\left(\frac{N_C}{3}\right)$ yielding $\omega_\Lambda \approx 217$ MeV which is only about 10% larger than the energy eigenvalue of the exact solution to eq. (3.5). Obviously, the energy eigenvalue ω_Λ is a non-linear function of flavor symmetry breaking. In particular, the observed relation $3\Gamma/2\Theta_K > \omega_0^2$ indicates that the perturbative treatment of flavor symmetry breaking is bound to fail⁵.

The quadratic equation (3.9) has a second solution: ω_Θ . It corresponds to strangeness $S = +1$ fluctuations and determines the mass difference between the Θ^+ pentaquark and the nucleon,

$$\omega_\Theta = \frac{1}{2} \left[\sqrt{\omega_0^2 + \frac{3\Gamma}{2\Theta_K}} + \omega_0 \right]. \quad (3.11)$$

Also this eigenmode evolves from the zero-mode. However, the Wess-Zumino term shifts ω_Θ to the positive continuum. We therefore expect a resonance structure in the phase shift around $\omega_\Theta = 396$ MeV and 617 MeV for the cases $m_K = m_\pi$ and $m_K \neq m_\pi$, respectively. These phase shifts are shown in fig. 1. Apparently no clear resonance structure is visible; the phase shifts hardly reach $\pi/2$ at the estimated energies. The absence of such a resonance has previously led to the conjecture and criticism that

⁵ The realistic case is even worse, as Γ is underestimated by about 50% in our approximation with $f_K = f_\pi$.

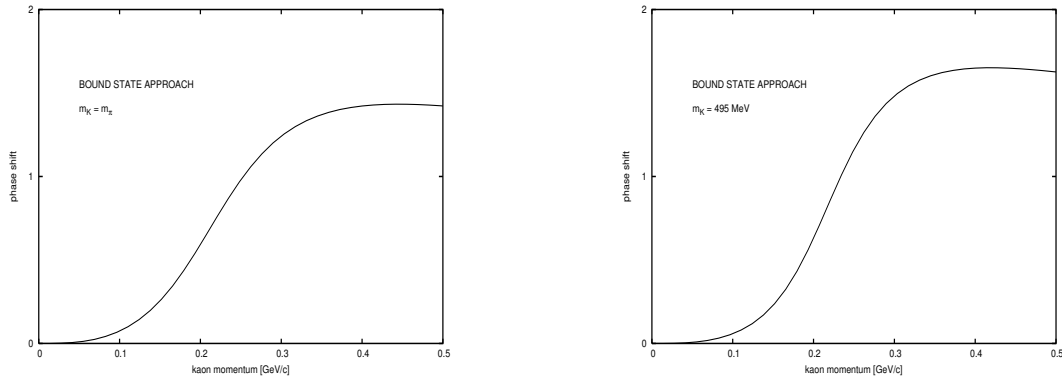


Fig. 1. The phase shift in the strangeness $S = +1$ channel for $m_K = m_\pi$ (left panel) and $m_K \neq m_\pi$ (right panel) as functions of the kaon momentum $k^2 = \omega^2 - m_K^2$ in the BSA.

there would not exist a bound pentaquark in the large- N_C limit [32] and that it would be a mere artifact of the RRA. The absence of pentaquarks in the BSA does not seem to be a result limited to the simple Skyrme model. For example, in the BSA to the NJL chiral quark soliton model a pentaquark bound state has not been observed either [55].

We have obtained the above equations for ω_A and ω_Θ by projecting the BSA equation (3.5) onto the rotational zero-mode. Hence they are exact when the fluctuations are constrained to the rotational subspace.

3.2 Fluctuations orthogonal to rigid rotations

Similar to the above study we eventually have to consider small amplitude fluctuations about the rotating soliton in the RVA, *i.e.* $\tilde{\eta}_\alpha$ in eq. (2.11) in sects. 5-8. In these sections we will show that the phase shifts extracted from $\tilde{\eta}$ and η are identical in the large- N_C limit, the ultimate test on the RVA. In contrast to the BSA fluctuations η_α explored so far, the $\tilde{\eta}_\alpha$ must be constrained to the subspace orthogonal to the rigid rotations of the hedgehog to avoid double-counting. These constraints follow directly from the *ansatz*, eq. (2.11) and we will derive them in the appendix. Here we will study the solutions to the equations of motion that arise when the BSA equations of motion (3.3) are supplemented by these constraints. We call these solutions $\tilde{\eta}$ and $\tilde{\Pi}$.

As noted above, the RVA Lagrangian contains terms that are linear in $\tilde{\eta}$ providing Yukawa couplings to rotational excitations of the soliton. The equation of motion for $\tilde{\eta}$ will then differ from the one for η in two aspects: i) the constraints and ii) exchanges of resonances induced by the Yukawa couplings. The important point here is obvious: supplementing the equation of motion for η by the constraints or omitting the Yukawa exchange terms from the equation for $\tilde{\eta}$ identically gives the one for the auxiliary field $\tilde{\eta}$. Since $\tilde{\eta}$ is obtained by removing the resonance pieces from $\tilde{\eta}$, it is the background contribution to the scattering amplitude. This strongly motivates the study of the auxiliary field $\tilde{\eta}$ as an intermediate step to construct the RVA fluctuations, $\tilde{\eta}$; even though the introduction of $\tilde{\eta}$ seems somewhat artificial at this moment.

The constraints only concern the P -wave channel and read

$$\begin{aligned} \Psi &= \int_0^\infty r^2 dr z(r) [\tilde{\Pi}(r) - \lambda(r)\tilde{\eta}(r)] = 0 \quad \text{and} \\ \chi &= \int_0^\infty r^2 dr z(r) M_K(r)\tilde{\eta}(r) = 0. \end{aligned} \quad (3.12)$$

We add these constraints to the Hamiltonian with global Lagrange multipliers α and β . From this we obtain the homogeneous equations of motion for the fluctuations $\tilde{\Pi}$ and $\tilde{\eta}$, that are orthogonal to the rotations of the soliton, as modifications to eqs. (3.3),

$$\begin{aligned} \tilde{\Pi}(r) &= [\omega M_K(r) - \lambda(r)] \tilde{\eta}(r) + \alpha M_K(r) z(r), \\ h^2 \tilde{\eta}(r) + \frac{\lambda^2(r)}{M_K(r)} &= \left[\omega - \frac{\lambda(r)}{M_K(r)} \right] \tilde{\Pi}(r) \\ &\quad - \alpha \lambda(r) z(r) - \beta M_K(r) z(r). \end{aligned} \quad (3.13)$$

We multiply the above equations by $z(r)$ from the left and integrate over the radial coordinate to determine the Lagrange multipliers using the constraints, eq. (3.12),

$$\begin{aligned} \alpha &= \int_0^\infty r^2 dr z(r) [2\lambda(r)] \tilde{\eta}(r) \quad \text{and} \\ \beta &= \omega_0 \alpha + (m_K^2 - m_\pi^2) \int_0^\infty r^2 dr z(r) \tilde{\eta}(r), \end{aligned} \quad (3.14)$$

since $\int r^2 dr z M_K z = 1$ and $\int r^2 dr z 2\lambda z = \omega_0$. We eliminate $\tilde{\Pi}$ from the second equation (3.13), substitute the above Lagrange multipliers and obtain the linear second-order integro-differential equation for $\tilde{\eta}$,

$$\begin{aligned} h^2 \tilde{\eta}(r) + \omega [2\lambda(r) - \omega M_K(r)] \tilde{\eta}(r) &= \\ - [2\lambda(r) - (\omega + \omega_0) M_K(r)] z(r) \int_0^\infty r'^2 dr' z(r') 2\lambda(r') \tilde{\eta}(r') & \\ + (m_K^2 - m_\pi^2) \int_0^\infty r'^2 dr' z(r') \tilde{\eta}(r'). \end{aligned} \quad (3.15)$$

Any solution $\tilde{\eta}$ to this equation automatically satisfies $\int r^2 dr z M_K \tilde{\eta} = 0$. This is easily confirmed by multiplying eq. (3.15) from the left by $z(r)$ and integrating over

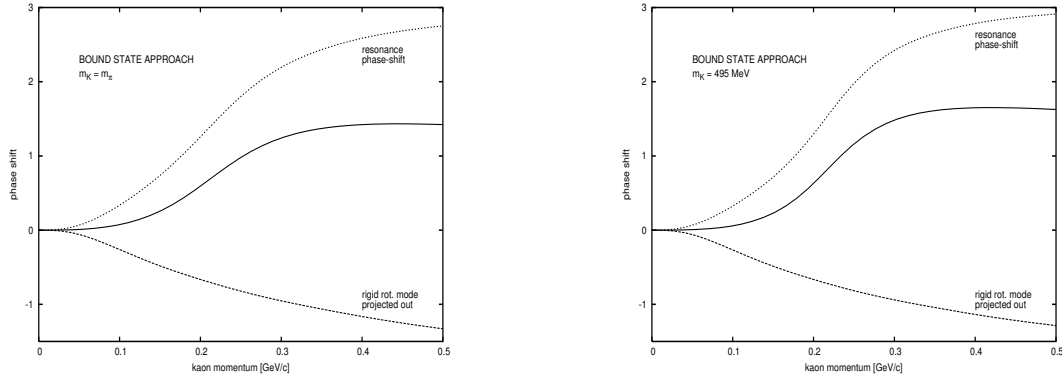


Fig. 2. The full lines are the same phase shifts as in fig. 1. The dashed lines are the background phase shifts in the strangeness $S = +1$ channel as functions of the kaon momentum $k^2 = \omega^2 - m_K^2$ as computed from the integro-differential equation (3.15). The dotted lines give their difference to be identified as the resonance phase shift. Left panel: $m_K = m_\pi$, right panel: $m_K \neq m_\pi$.

the radial coordinate. Obviously this integro-differential equation projects eq. (3.5) on the subspace orthogonal to the rigid rotations of the hedgehog. As a result, the integro-differential equation does *not* have a bound-state solution to be associated with the Λ -hyperon. In fig. 2 the dashed lines are the phase shifts $\bar{\delta}$ for strangeness $S = +1$ as computed from the integro-differential equation (3.15). This phase shift is quite shallow with an indication of a small repulsive potential. As explained at the beginning of this subsection, $\bar{\eta}$ parameterizes the background part. That is, $\bar{\delta}$ is the *background* phase shift in the strangeness $S = +1$ channel. On the other hand, the total phase shift, δ (cf. fig. 1) is the sum of the *resonance* phase shift, δ_R , and the background phase shift,

$$\delta_R(k) = \delta(k) - \bar{\delta}(k). \quad (3.16)$$

We also display this resonance phase shift in fig. 2. Obviously, it exhibits a clear resonance structure as $\delta_R(k)$ smoothly passes through $\pi/2$. Thus, we reach a central result of our studies: Once the background phase shift is properly identified and subtracted, chiral soliton models indeed predict a distinct resonance structure in the strangeness $S = +1$ channel even and in particular for $N_C \rightarrow \infty$. In sect. 5 we will show that in the limit $N_C \rightarrow \infty$ it is exactly δ_R that arises from the exchange of the Θ^+ pentaquark (and the Λ -hyperon). Figure 2 suggests that the resonance is quite broad. In sect. 6 we will discuss that finite- N_C effects are substantial in sharpening this resonance. Note also from fig. 2 that the position at which $\delta_R(k)$ passes $\pi/2$ is somewhat lower than predicted by the estimate, eq. (3.11). In sect. 6 we will explain that this pole shift originates from the well-known effect of iterated resonance exchanges. Actually this is standard in the scattering theory and does not alter the observation that ω_Θ is the “bare” resonance or pole mass.

3.3 Hyperfine splitting

Having established the existence of states in both the $S = -1$ (as bound state) and $S = +1$ (as resonance) channels we would like to complete this section discussing the

hyperfine splitting in the BSA for later comparison with the RRA. Even more, we have seen that the corresponding kaon wave functions are dominated by the zero-mode component, $z(r)$. *A posteriori* this justifies the estimates for the mass differences to the nucleon, eqs. (3.10) and (3.11). These estimates concern the large- N_C limit and are thus degenerate in spin and isospin. To lift this degeneracy, states with good spin and isospin need to be constructed in the framework of the BSA. For the $S = -1$ channel this has been performed in ref. [49] and we will generalize it to also apply for $S = +1$. Starting point is the parameterization, eq. (2.10) with the zero-mode wave function substituted for η_α in the P -wave, cf. eq. (3.2). Spin and isospin states are then generated by canonical quantization of the $SU(2)$ collective coordinates, A_2 . Substituting this parameterization into the action adds

$$\begin{aligned} \Delta L = & \frac{\Theta_\pi}{2} \vec{\Omega}^2 + i(2\Theta_K - \Theta_\pi) \vec{\Omega} \\ & \cdot \left[\left(\frac{dK(t)}{dt} \right)^\dagger \vec{\tau} K(t) - K^\dagger(t) \vec{\tau} \frac{dK(t)}{dt} \right] \\ & - \frac{N_C}{2} \vec{\Omega} \cdot K^\dagger(t) \vec{\tau} K(t) \end{aligned} \quad (3.17)$$

to the Lagrange function [49]. Here $\sqrt{2}K(t) = \int d\omega \chi(\omega) e^{i\omega t}$ contains the amplitudes for the kaonic states in the $S = \pm 1$ channels. The dependence on the collective coordinates is parameterized by the angular velocities

$$A_2^\dagger(t) \frac{dA_2(t)}{dt} = \frac{i}{2} \vec{\tau} \cdot \vec{\Omega}. \quad (3.18)$$

In addition, the moment of inertia for spatial rotations [14] appears,

$$\begin{aligned} \Theta_\pi = & \frac{2}{3} \int d^3r \sin^2 F \left[f_\pi^2 + \frac{1}{\epsilon^2} \left(F'^2 + \frac{\sin^2 F}{r^2} \right) \right] = \\ & 5.115 \text{ GeV}^{-1} \frac{N_C}{3}. \end{aligned} \quad (3.19)$$

Treating this additional term perturbatively modifies the Hamiltonian by

$$\Delta H = \frac{1}{2\Theta_\pi} \left[\vec{J} - \frac{i}{2} \left(1 - \frac{\Theta_\pi}{2\Theta_K} \right) \times (\Pi^\dagger \vec{\tau} K - K^\dagger \vec{\tau} \Pi) + \frac{N_C \Theta_\pi}{4\Theta_K} K^\dagger \vec{\tau} K \right]^2, \quad (3.20)$$

where $J_i = \frac{\partial \Delta L}{\partial \Omega_i}$ and Π is conjugate to K^\dagger . The computation of matrix elements of ΔH is standard [49] and yields the *hyperfine* contributions to the masses of baryons with strangeness $S = \pm 1$,

$$\Delta M_S = \frac{1}{2\Theta_\pi} \left[c_S J(J+1) + (1-c_S) I(I+1) + \frac{3}{4} c_S (c_S - 1) \right], \quad (3.21)$$

with J and I being the spin and isospin quantum numbers of the considered baryon, respectively. The hyperfine parameters

$$c_- = 1 - \frac{4\Theta_\pi \omega_\Lambda}{8\Theta_K \omega_\Lambda + N_C} \quad \text{and} \quad c_+ = 1 - \frac{4\Theta_\pi \omega_\Theta}{8\Theta_K \omega_\Theta - N_C} \quad (3.22)$$

are $\mathcal{O}(N_C^0)$ so that the coefficient $1/\Theta_\pi$ in eq. (3.21) ensures $\lim_{N_C \rightarrow \infty} \Delta M_S = 0$. We remark that the expressions in eqs. (3.10), (3.11) and (3.22) are the BSA predictions for fluctuations η in the subspace of rigid rotations.

To this end, the mass differences of the Λ -hyperon and the Θ^+ pentaquark with respect to the nucleon read

$$M_{\Lambda, \Theta} - M_N = \omega_{\Lambda, \Theta} + \Delta M_S - \frac{3}{8\Theta_\pi}. \quad (3.23)$$

Similar relations can be easily written down for other $S = \pm 1$ baryon states. The last term in eq. (3.21) is often omitted as it contains terms that are quartic in the fluctuating field and the calculation is not completely carried out at this order. The comparison with the RRA suggests that these quartic terms contribute $9/8\Theta_K$ to the mass of the $S = 1$ baryons. This is not negligible for $N_C = 3$ and clearly shows that subleading contributions *must* be considered for predictions to be reliable. These subleading contributions can be evaluated within the RRA that we will discuss in the next section.

4 Rigid-rotator approach (RRA)

In this section we will explain how baryon states emerge as soliton excitations that are members of flavor $SU(3)$ representations. They are obtained from canonical quantization of the collective coordinates A_3 in the *ansatz*, eq. (2.11). For this purpose we do not need to consider the fluctuations $\tilde{\eta}_\alpha$ and relegate the discussion of the coupling between A_3 and $\tilde{\eta}_\alpha$ to the following section. The resulting Lagrangian for the collective coordinates reads [30]

$$L = -M + \frac{1}{2} \Theta_\pi \Omega_i^2 + \frac{1}{2} \Theta_K \Omega_\alpha^2 - \frac{N_C}{2\sqrt{3}} \Omega_8 - \frac{1}{2} \Gamma (1 - D_{88}). \quad (4.1)$$

The soliton mass M is the functional which upon minimization determines the chiral angle $F(r)$. Its particular form [14] is of no relevance here because we will only consider mass differences. The moments of inertia, Θ_π, Θ_K , and the symmetry breaking parameter, Γ , have already been introduced in the previous section, cf. eqs. (3.6), (3.9) and (3.19). In eq. (4.1) also angular velocities for rotations into strangeness direction appear. These are straightforward generalizations of the definition (3.18) to A_3

$$A_3^\dagger(t) \frac{dA_3(t)}{dt} = \frac{i}{2} \lambda_a \Omega_a. \quad (4.2)$$

Finally the symmetry breaking part of the action, eq. (2.6), causes the explicit dependence on the collective coordinates. This is conveniently displayed with the help of their adjoint representation,

$$D_{ab}(A_3) = \frac{1}{2} \text{tr} \left[\lambda_a A_3 \lambda_b A_3^\dagger \right]. \quad (4.3)$$

We note that in more complicated soliton models (*e.g.* with vector mesons or chiral quarks) the symmetry breaking part of L has a richer structure [31] which eventually leads to better agreement between predicted and observed baryon spectra. However, we presently compare two different treatments (BSA *vs.* RRA) of the *same* model and thus other symmetry breaking terms are of no relevance to us. For quantization we compute the conjugate momenta,

$$R_a = -\frac{\partial L}{\partial \Omega_a} = \begin{cases} -\Theta_\pi^2 \Omega_i = -J_i, & a = i = 1, 2, 3, \\ -\Theta_K \Omega_\alpha, & a = \alpha = 4, \dots, 7, \\ \frac{N_C}{2\sqrt{3}}, & a = 8. \end{cases} \quad (4.4)$$

The quantization prescription then demands the commutation relations $[R_a, R_b] = -if_{abc} R_c$ with f_{abc} being the antisymmetric-structure constants of $SU(3)$. That is, the R_a are the generators of the intrinsic $SU(3)$ group. The hedgehog structure of the classical soliton ensures that $J_i = -R_i$, ($i = 1, 2, 3$), is the total spin operator. The Wess-Zumino term, eq. (2.5), plays a decisive role. It is non-local and only linear in the time derivative such that it contributes the term linear in Ω_8 to the collective-coordinate Lagrangian. Therefore, the corresponding conjugate momentum is constrained to $R_8 = \frac{N_C}{2\sqrt{3}}$ and we require a Lagrange multiplier, μ , for this constraint when constructing the Hamiltonian for the collective coordinates by Legendre transformation from eq. (4.1)

$$H = M + \frac{1}{2\Theta_\pi} R_i^2 + \frac{1}{2\Theta_K} R_\alpha^2 + \frac{1}{2} \Gamma (1 - D_{88}) - \mu \left(R_8 - \frac{N_C}{2\sqrt{3}} \right). \quad (4.5)$$

Demanding that L can be obtained from H by the inverse Legendre transformation requires to set $\mu = \Omega_8$. Since $[H, R_8] = 0$ this angular velocity remains undetermined and different choices merely imply different gauges. This vanishing commutator also suggests that the constraint can be implemented on the eigenstates of H . Some of the discussion on finding these eigenstates for arbitrary N_C is already contained in the literature [49]. Here we focus

on the comparison between BSA and RRA for the $S = +1$ pentaquarks, which is new. So is the non-perturbative investigation of symmetry breaking effects in the RRA for arbitrary N_C ⁶.

We concentrate on the case of vanishing symmetry breaking, $\Gamma = 0$ for a first discussion on finding the eigenstates of the collective-coordinate Hamiltonian, eq. (4.5). For $\Gamma = 0$ the eigenstates of H are members of definite irreducible representations of $SU(3)$ that are characterized by the labels (p, q) . The eigenvalues of H can then be computed from the eigenvalue of the quadratic Casimir operator of $SU(3)$,

$$C_2 = \langle R_a^2 \rangle = \frac{1}{3} (p^2 + pq + q^2) + p + q. \quad (4.6)$$

The constraint on R_8 only allows representations with

$$p + 2q = N_C + 3t, \quad (4.7)$$

where $t = 0, 1, 2, \dots$ refers to Biedenharn's triality [2]. The physical hypercharge for baryons is generalized to $Y = \frac{N_C}{3} + S$. Note that because of $Y_{\max} = \frac{N_C}{3} + t$, this triality equals the largest strangeness quantum number $S_{\max} = t$ contained in the representation (p, q) . The eigenstates of H are most conveniently arranged according to increasing t . For $t = 0$ they are

$$(p, q) = \left(2J, \frac{N_C - 2J}{2} \right), \quad (4.8)$$

where $J = \frac{1}{2}, \frac{3}{2}, \dots, \frac{N_C}{2}$ are the allowed spin eigenvalues, *i.e.* $\langle R_i^2 \rangle = J(J+1)$. From eq. (4.6) we compute the corresponding energies,

$$E_0 = M + \frac{J(J+1)}{2\Theta_\pi} + \frac{N_C}{4\Theta_K}, \quad (4.9)$$

which coincides with the $SU(2)$ mass formula [14] up to an additive constant for nucleon and Δ -type states. For $t = 1$ we find

$$(p, q) = \left(2J \mp 1, \frac{N_C + 3}{2} - \frac{2J \mp 1}{2} \right), \\ E_{\mp} = M + \frac{J(J+1)}{2\Theta_\pi} + \frac{2N_C + 4 \mp (2J + 1)}{4\Theta_K}, \quad (4.10)$$

with $J = \frac{1}{2}, \frac{3}{2}, \dots, \frac{N_C}{2} + 1$. In particular, the lowest-lying multiplets with spin $J = \frac{1}{2}$ and $J = \frac{3}{2}$ are

$$J = \frac{1}{2} : (p, q) = \left(1, \frac{N_C - 1}{2} \right), \left(0, \frac{N_C + 3}{2} \right), \dots = \\ \text{"8"}, \text{"}\overline{\mathbf{10}}\text{"}, \dots \\ J = \frac{3}{2} : (p, q) = \left(3, \frac{N_C - 3}{2} \right), \left(2, \frac{N_C + 1}{2} \right), \dots = \\ \text{"}\mathbf{10}\text{"}, \text{"}\mathbf{27}\text{"}, \dots, \quad (4.11)$$

⁶ A simplified treatment of symmetry breaking to only leading order in perturbation theory gives results consistent with ours [56].

where the expressions to the furthest right refer to the dimensionalities of the representations for $N_C = 3$. The above assignments are unique in soliton models and so are the flavor quantum numbers attributed to the lowest-lying baryon states within the multiplets. Essentially this is inferred from the fact that these assignments minimize the energy functional. For example, for large N_C the symmetry breaking term contributes

$$\frac{\Gamma}{2} \langle 1 - D_{88} \rangle \rightarrow \frac{\Gamma}{2} \left(1 - \frac{3}{N_C} Y \right) = -\frac{3\Gamma}{2N_C} S, \quad (S \leq t), \quad (4.12)$$

to the energy functional if symmetry breaking was treated in first-order perturbation. For finite N_C this generalizes to the fact that symmetry breaking causes the masses of baryons within a multiplet to decrease with increasing strangeness. For $t = 0$ we therefore obtain the lightest $J = \frac{1}{2}$ baryon, the "nucleon" with zero strangeness and isospin $I = \frac{1}{2}$ and the lightest $J = \frac{3}{2}$ baryon, the " Δ " with zero strangeness and isospin $I = \frac{3}{2}$. For $t = 1$ the lowest-lying "pentaquarks" with $J = \frac{1}{2}$ and $J = \frac{3}{2}$ carry strangeness $S = +1$ and isospin $I = 0$ and $I = 1$, respectively. Of course, for finite symmetry breaking mixing with higher-dimensional multiplets occurs (see below). Although such non-linear effects may not be omitted, the spin-flavor assignments are unaffected.

The lowest-lying pentaquarks with $S = +1$ dwell in $t = 1$ representations and the two mass formulas (4.10) refer to the spin-isospin relations $I = J \mp \frac{1}{2}$. It is straightforward to derive the corresponding energies

$$E_{\mp} = M + \frac{J(J+1)}{2\Theta_\pi} + \frac{N_C}{2\Theta_K} \\ + \frac{1}{2\Theta_K} \left[I(I+1) - J(J+1) + \frac{9}{4} \right]. \quad (4.13)$$

The terms $\mathcal{O}(1/N_C)$ exactly match⁷ those in eq. (3.21) when equating the hyperfine energies that are related to expressions quartic in the fluctuations to $\frac{9}{8\Theta_K}$. Moreover, we may compute the mass difference between the nucleon (in " $\mathbf{8}$ ") and pentaquarks with $I = 0$ and $J = \frac{1}{2}$ (in " $\overline{\mathbf{10}}$ "),

$$E_{\overline{\mathbf{10}}} - E_{\mathbf{8}} = \frac{N_C + 3}{4\Theta_K}, \quad (4.14)$$

which coincides with ω_Θ as $N_C \rightarrow \infty$, as it should. Thus, we conclude that the BSA and RRA are consistent when flavor symmetry breaking is omitted. Note that this mass difference acquires a factor 2 for $N_C = 3$.

We will next consider the physical case of non-zero flavor symmetry breaking. As a first measure a perturbative treatment of flavor symmetry breaking seems tempting because it reproduces the Gell-Mann–Okubo mass relations [57,58] in the RRA for $N_C = 3$ [30]. However, we already argued in the context of eqs. (3.10) and (3.11) that the perturbation series in Γ does not converge for the BSA. We recall that many of the pentaquark studies

⁷ Without symmetry breaking we have $\omega_\Theta = N_C/4\Theta_K$ and thus $1 - c_+ = \Theta_\pi/\Theta_K$.

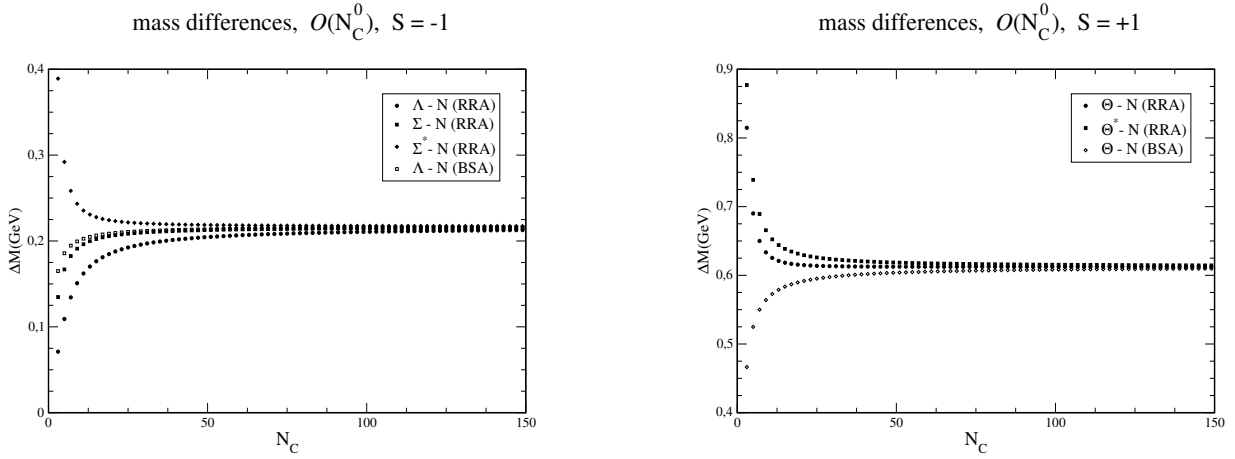


Fig. 3. Mass differences at $\mathcal{O}(N_C^0)$ computed for $\epsilon = 4.25$ in the Skyrme model as functions of N_C . In the RRA they are the corresponding differences of the eigenvalues in eq. (4.15), while in the BSA they are extracted from eq. (3.23). Left panel: $\Delta S = -1$; right panel: $\Delta S = +1$.

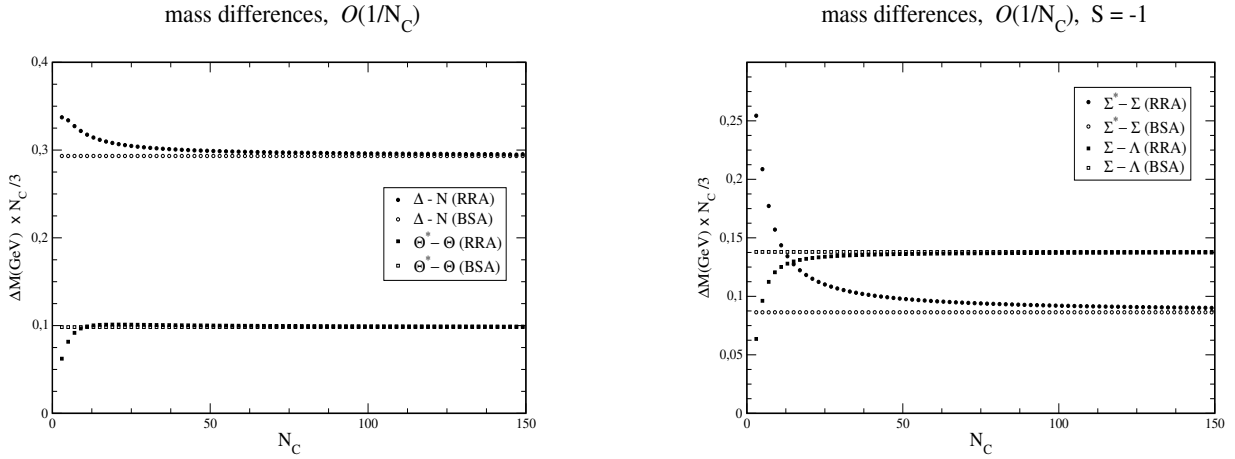


Fig. 4. Mass differences at $\mathcal{O}(1/N_C)$ computed for $\epsilon = 4.25$ in the Skyrme model as functions of N_C . Left panel: baryons with strangeness $S = 0, 1$; right panel: $S = -1$. See also the caption of fig. 3.

in the RRA treated flavor symmetry breaking perturbatively (at best to second order) [10,41]⁸. This seems at odds with the BSA. Also, the Gell-Mann–Okubo relations are not reproduced in the BSA [31]. Hence we have to solve the eigenvalue problem

$$\left\{ M + \frac{J(J+1)}{2\Theta_\pi} + \frac{R_\alpha^2}{2\Theta_K} + \frac{1}{2}\Gamma(1 - D_{88}) \right\} \Psi = E\Psi$$

together with $R_8\Psi = \frac{N_C}{2\sqrt{3}}\Psi$ (4.15)

(numerically) exactly. For $N_C = 3$ this has been accomplished some time ago [60] and frequently used since then for the computation of a variety of baryon properties, cf. refs. [59,61,62]. This can straightforwardly be generalized by putting $Y_R = 2R_8/\sqrt{3} = N_C/3$ in eq. (2.18) of ref. [60], together with the N_C dependences of Θ_π , cf. eq. (3.6) and Γ (paragraph after eq. (3.10)). Equivalently, the operator

⁸ In the case of pentaquarks, symmetry breaking was treated exactly in refs. [5,6,38,59].

D_{88} may be diagonalized in the space of $SU(3)$ representations with $R_8\Psi = \frac{N_C}{2\sqrt{3}}\Psi$ using $SU(3)$ Clebsch-Gordan coefficients. The only condition on N_C is that it must be taken odd. In sects. 7 and 8 we will employ the resulting wave functions to compute matrix elements of collective-coordinate operators for arbitrary odd N_C and non-zero symmetry breaking.

For the previously mentioned parameters we present the resulting mass differences in figs. 3 and 4. In fig. 3 we concentrate on mass differences that scale like $\mathcal{O}(N_C^0)$, *i.e.* between baryons whose strangeness quantum numbers differ by one unit. As $N_C \rightarrow \infty$ the BSA for fluctuations in the rotational subspace predicts ω_Λ and ω_Θ for those mass differences regardless of spin and isospin. Obviously this is matched by the RRA. In fig. 4 we display the mass differences that scale like $\mathcal{O}(1/N_C)$, *i.e.* we compute the hyperfine splitting in both approaches. In doing so we only consider combinations for which in the BSA the ordering ambiguities and omissions from $\mathcal{O}(\eta_\alpha^4)$ terms cancel, *i.e.* baryons with identical strangeness. Again, perfect

agreement between the two approaches is observed for $N_C \rightarrow \infty$. However, at $N_C = 3$ sizable $1/N_C$ corrections occur that cannot be accounted for by the BSA.

A couple of further comments are in order. First, when computing matrix elements we observe that, cf. eq. (4.12),

$$\frac{\Gamma}{2} \langle 1 - D_{88} \rangle = \mathcal{O}(N_C^0), \quad (4.16)$$

that is, the net effect of flavor symmetry breaking in the RRA is an $\mathcal{O}(N_C^0)$ contribution to the baryon mass. Again, this is consistent with the BSA where flavor symmetry breaking enters the $\mathcal{O}(N_C^0)$ equation of motion for the kaon fluctuations (3.5). Second, the literature contains claims that the Δ -nucleon mass difference would scale like $\mathcal{O}(N_C^0)$ once symmetry breaking is incorporated in the RRA [11]. The actual calculation, displayed in fig. 4, shows that this is not correct. As in two extreme cases, zero and infinite⁹ symmetry breaking, this mass difference scales like $\mathcal{O}(1/N_C)$. The physical value, $m_K = 495$ MeV is intermediate to these extreme cases and it thus comes to no surprise that this mass difference vanishes as $N_C \rightarrow \infty$.

The second remark concerns the eventual restriction of collective coordinates to only dynamical zero-modes. For zero symmetry breaking ($\Gamma = 0$) the rigid-rotation eigenstates of eq. (4.15) are states in the **8**, **10**, **$\overline{10}$** , **27**, . . . dimensional representations of flavor $SU(3)$. Though the antikaon fluctuations are zero frequency solutions in the BSA, the kaon fluctuations are not due to the Wess-Zumino term, *i.e.* the latter are not really dynamical zero-modes. Therefore, it has been suggested to introduce collective coordinates *only* for the antikaonic zero-modes in a manner that only the octet and decuplet would be treated collectively [34]. Such an approach seems complicated because constraints on both the fluctuations and the collective coordinates are required. Putting the question of feasibility aside, there is no advantage in doing so to study the Θ^+ pentaquark since such a treatment essentially corresponds to the BSA and prevents the incorporation of finite N_C effects. The restriction to the octet and decuplet seems even more problematic in view of flavor symmetry breaking ($\Gamma \neq 0$) as it mixes baryon states from different $SU(3)$ representations [61]. In particular the nucleon wave function contains admixture of the nucleon-type state in the **$\overline{10}$** . Thus, the latter may not be omitted.

To round up this section on the RRA we note that other approaches to quantize chiral solitons in flavor $SU(3)$, like the slow-rotator [63] or the breathing-mode [37] approaches, are essentially extensions of the RRA.

So far we have established the relation between rotational excitations of the soliton and the bound states in the large- N_C limit. In order to study the decay width of the Θ^+ pentaquark we need to introduce fluctuations by extending the RRA to the RVA.

5 Collective rotations and vibrations

We will now include the kaon fluctuations in the RVA according to the *ansatz* (2.11) and compare it to results for the fluctuations in the BSA, sect. 3. The flavor rotating hedgehog $A_3(t)U_0(\vec{x})A_3^\dagger(t)$ is *not* a solution to the field equations. Hence the expansion of the action with respect to $\tilde{\eta}$ has a linear term. This linear term gives rise to Yukawa couplings like $KN\Lambda$ and $KN\Theta$. As a consequence, the KN phase shift in the isospin $I = 0$ channel¹⁰ has contributions from the (iterated) exchange of Λ and Θ . Technically, these exchange contributions emerge in the equation of motion for $\tilde{\eta}$ as a separable potential of the form “(Yukawa interaction) \times (propagator) \times (Yukawa interaction)”. The reader may want to peek ahead to eq. (5.10) for the explicit expression. We have already noted that the background field $\bar{\eta}$, that we have computed in sect. 3, obeys an equation of motion that is obtained from the one for $\tilde{\eta}$ by omitting the Yukawa couplings. That is, the respective equations of motion differ by this separable potential and we can straightforwardly construct $\tilde{\eta}$ from $\bar{\eta}$ using standard Lippmann-Schwinger equation techniques. This will make the role of exchanged resonances very transparent. For convenience we list the notation for the involved radial functions:

- $\eta(r)$: solution to eq. (3.5) in the BSA,
- $\bar{\eta}(r)$: solution to the integro-differential equation, (3.15), with constraints,
- $\tilde{\eta}(r)$: solution to the integro-differential equation, (5.10), with constraints and Yukawa exchange.

We will establish two central results in the large- N_C limit:

- a) We compute the RVA fluctuations $\tilde{\eta}$ by supplementing the equations of motion for $\bar{\eta}$ with the effects due to Λ and Θ exchanges. We compute the phase shifts for $\tilde{\eta}$ and show that they are *identical* to those computed from η in the BSA, eq. (2.10).
- b) We separate the Yukawa exchange contributions to the scattering amplitude and find the corresponding phase shift to *equal* the resonance phase shift displayed in fig. 2.

In doing so we need to identify the Yukawa couplings by computing the terms linear in $\tilde{\eta}$. Note that in the action all linear terms must occur as isosinglets. Then there are two sources for Yukawa terms: 1) the flavor symmetric piece $\Gamma_{\text{SK}} + \Gamma_{\text{WZ}}$ has couplings to the angular velocities Ω_a , 2) the symmetry breaking piece has a contraction between D_{8a} and $\tilde{\eta}_a$. For the moment we will only consider the first case and relegate the discussion of flavor symmetry breaking to sect. 7.

In the flavor symmetric case there are already many rotation-vibration coupling terms linear and quadratic in Ω_a . In order to keep the calculation feasible we keep only those terms that survive in the large- N_C limit. The so-selected Yukawa coupling is then treated to *all* orders

¹⁰ The $I = 1$ channel, which we discuss in the last paragraph of sect. 8, has Σ^* and Θ^* exchange contributions.

⁹ This is the $SU(2)$ version of the chiral soliton model.

in N_C . Admittedly, this procedure is not completely consistent and the neglected couplings are not necessarily small at $N_C = 3$. However, a major purpose of the present investigation is to show the equivalence with the BSA. For this purpose it completely suffices to only consider the leading N_C couplings. In order to isolate these terms we need to discuss the N_C scaling of the angular velocities Ω_a to identify those Yukawa terms that survive in the large- N_C limit. This is straightforward for $a = 1, \dots, 7$ as we simply invert eq. (4.4),

$$\begin{aligned} \sum_{i=1}^3 \Omega_i^2 &= \frac{J(J+1)}{\Theta_\pi^2} \Rightarrow \Omega_i = \mathcal{O}(1/N_C), \\ \text{i.e. } R_i &= \mathcal{O}(N_C^0), \\ \sum_{\alpha=4}^7 \Omega_\alpha^2 &= \frac{C_2 - J(J+1) - N_C^2/12}{\Theta_K^2} \Rightarrow \Omega_\alpha = \mathcal{O}(1/\sqrt{N_C}), \\ \text{i.e. } R_\alpha &= \mathcal{O}(\sqrt{N_C}), \end{aligned} \quad (5.1)$$

as the moments of inertia Θ_π and Θ_K are $\mathcal{O}(N_C)$. In the large- N_C expansion Ω_i ($i = 1, 2, 3$) is thus subleading to Ω_α ($\alpha = 4, \dots, 7$) and may be omitted. As will be shown in the appendix, the angular velocity Ω_8 may be absorbed in a suitable redefinition of the energy scale,

$$\omega \longrightarrow \omega + \frac{\sqrt{3}}{2} \Omega_8, \quad (5.2)$$

where ω is the frequency of the kaon fluctuation, cf. eq. (3.2). Thus, to compute the $\Omega\tilde{\eta}$ Yukawa coupling we only need to consider the kaonic angular velocities $\Omega_\alpha = -R_\alpha/\Theta_K$ with $\alpha = 4, \dots, 7$. This lengthy calculation described in the appendix not only requires to extract the terms in the action that are linear in $\tilde{\eta}$ but also to consider the constraints, eq. (3.12), because they contain pieces that are of the same linear order in $\tilde{\eta}$. Having extracted these linear terms we are unfortunately not yet in the position to compute matrix elements of the interaction Hamiltonian (5.5) between Θ and KN states in the space of the collective coordinates. The reason is that $\tilde{\eta}$ parameterizes the kaon field in the intrinsic frame. A collective flavor rotation relates the intrinsic fluctuations to those in the laboratory frame, that are observed in kaon nucleon scattering,

$$\tilde{\xi}_a = D_{ab} \tilde{\eta}_b, \quad (5.3)$$

with $a, b = 1, \dots, 8$. In the large- N_C limit, $D_{\alpha i}$ and $D_{\alpha 8}$, the couplings to the intrinsic pions and η , respectively, are suppressed. Hence we approximate

$$\tilde{\xi}_\alpha = D_{\alpha\beta} \tilde{\eta}_\beta, \quad (5.4)$$

with $\alpha, \beta = 4, \dots, 7$. Finally, the resulting interaction Hamiltonian reads

$$\begin{aligned} H_{\text{int}} &= \frac{2f_\pi}{\Theta_K} d_{i\alpha\beta} D_{\gamma\alpha} R_\beta \\ &\times \int d^3r \sin \frac{F(r)}{2} [2\lambda(r) - \omega_0 M_K(r)] \hat{x}_i \tilde{\xi}_\gamma(\vec{x}, t), \end{aligned} \quad (5.5)$$

where $d_{i\alpha\beta}$ are the symmetric structure constants of $SU(3)$. The pieces involving $\omega_0 = N_C/4\Theta_K$ and $\lambda(r)$ stem from Γ_{SK} and Γ_{WZ} , respectively. From eqs. (2.7), (5.1) we easily verify that $H_{\text{int}} = \mathcal{O}(N_C^0)$.

We need to compute matrix elements of the interaction Hamiltonian, eq. (5.5), between a kaon nucleon state coupled to good spin (J) and isospin (I) and a collective excitation with the same quantum numbers. We denote the operator that annihilates a kaon in the laboratory frame with isospin projection $i = \pm\frac{1}{2}$ and orbital angular momentum (ℓ, m) by $a_{\ell, m}^{(i)}$ and use the Wigner-Eckart theorem to reduce the matrix elements of H_{int} to those computed in the space of the collective coordinates,

$$\begin{aligned} \sum_{i=\pm\frac{1}{2}} \langle \Theta | a_{1, m}^{(i)} d_{m\alpha\beta} D_{\gamma(i)\alpha} R_\beta | (KN)_{I=0, J=\frac{1}{2}} \rangle = \\ \sqrt{6} \langle \Theta^+ \uparrow | d_{3\alpha\beta} D_{+\alpha} R_\beta | n \uparrow \rangle, \end{aligned} \quad (5.6)$$

where $\gamma(\pm\frac{1}{2})$ maps the isospin projection onto the flavor indices, $D_{\pm a} = D_{4a} \pm iD_{5a}$. The arrow indicates the spin projection $J_3 = \frac{1}{2}$. In what follows we will always adopt this value without explicit mention. Orbital angular momentum $\ell = 1$ is projected onto by \hat{x}_i in H_{int} . Of course, this is nothing but the statement that only the P -wave channel is affected by the rigid rotations. A similar reduction formula holds for the matrix element of H_{int} between final Λ and initial antikaon nucleon states. In the flavor symmetric case the relevant matrix elements of the collective-coordinate operators are

$$\begin{aligned} \langle \Theta^+ | d_{3\alpha\beta} D_{+\alpha} R_\beta | n \rangle = \\ \frac{N_C + 1}{4} \sqrt{\frac{N_C + 1}{2(N_C + 3)(N_C + 7)}} =: X_\Theta \sqrt{\frac{N_C}{32}}, \\ \langle \Lambda | d_{3\alpha\beta} D_{-\alpha} R_\beta | p \rangle = \\ -\frac{1}{2} \frac{N_C + 5}{(N_C + 7)\sqrt{2N_C + 6}} =: X_\Lambda \sqrt{\frac{N_C}{32}}. \end{aligned} \quad (5.7)$$

For the matrix elements in eq. (5.7) we have factored the large- N_C value for later convenience,

$$X_\Theta = \sqrt{\frac{(N_C + 1)^3}{N_C(N_C + 3)(N_C + 7)}} = \begin{cases} 1, & N_C \rightarrow \infty, \\ \frac{4}{3\sqrt{5}}, & N_C = 3, \end{cases} \quad (5.8)$$

$$X_\Lambda = -\frac{2(N_C + 5)}{(N_C + 7)\sqrt{N_C(N_C + 3)}} = \begin{cases} 0, & N_C \rightarrow \infty, \\ -\frac{4\sqrt{2}}{15}, & N_C = 3. \end{cases}$$

Only the Θ^+ Yukawa coupling survives as $N_C \rightarrow \infty$. Hence there is only one intermediate state in that limit. For the physical value, $N_C = 3$, the collective-coordinate part of the Λ Yukawa coupling constant is actually larger in magnitude than that of Θ^+ .

Obviously this explicit derivation of the interaction Hamiltonian yields only a *single* $SU(3)$ structure (*i.e.* $d_{i\alpha\beta} \hat{x}_i D_{a\alpha} R_\beta$) for the Θ - N transition. This is to be contrasted with earlier studies [10, 11, 41, 42] who “invented” three structures, even in the flavor symmetric case.

We want to add the particle exchanges induced by H_{int} to the differential equation, (3.15) for the constrained fluctuations, $\bar{\eta}$ (still omitting symmetry breaking). The propagators are $(\omega_\Theta - \omega)^{-1}$ and $(\omega_\Lambda + \omega)^{-1}$ for Θ and Λ , respectively, because the differential equations for kaons and antikaons are related by $\omega \leftrightarrow -\omega$. Thus the Yukawa terms induce the separable potential

$$\frac{|\langle \Theta | H_{\text{int}} | (KN)_{I=0} \rangle|^2}{\omega_\Theta - \omega} + \frac{|\langle \Lambda | H_{\text{int}} | (KN)_{I=0} \rangle|^2}{\omega_\Lambda + \omega}. \quad (5.9)$$

Note that these matrix elements concern the T -matrix elements in the laboratory frame. However, we observe from eq. (3.1) that for the Θ^+ channel the laboratory and intrinsic T -matrix elements are identical. Hence we may add the exchange potential, eq. (5.9), in the intrinsic frame. We then end up with the modified integro-differential equation

$$\begin{aligned} h^2 \bar{\eta}(r) + \omega [2\lambda(r) - \omega M_K(r)] \bar{\eta}(r) = \\ -z(r) \left[\int_0^\infty r'^2 dr' z(r') 2\lambda(r') \bar{\eta}(r') \right] \\ \times \left[2\lambda(r) - (\omega + \omega_0) M_K(r) - \omega_0 \left(\frac{X_\Theta^2}{\omega_\Theta - \omega} + \frac{X_\Lambda^2}{\omega} \right) \right] \\ \times (2\lambda(r) - \omega_0 M_K(r)) \end{aligned} \quad (5.10)$$

in the flavor symmetric case. The particular coefficient, ω_0 of the Yukawa contribution will become clear when we discuss the width of Θ^+ in the next section. The Yukawa contribution to the integro-differential equation is orthogonal to the zero-mode,

$$\int_0^\infty r^2 dr z(r) [2\lambda(r) - \omega_0 M_K(r)] z(r) = 0 \quad (5.11)$$

and thus any solution to eq. (5.10) still satisfies the second constraint in eq. (3.12) which we subsequently employ to simplify the integral.

The essential observation is that eq. (5.10) has a simple solution as $N_C \rightarrow \infty$ ($X_\Theta = 1$ and $X_\Lambda = 0$),

$$\bar{\eta}(r) = \eta(r) - az(r) \quad \text{with} \quad a = \int_0^\infty dr r^2 z(r) M_K(r) \eta(r). \quad (5.12)$$

Here $\eta(r)$ is the solution to the unconstrained equation (3.5) in the BSA. The radial function, $z(r)$ associated with the zero-mode is localized in space. Thus the phase shifts of η and $\bar{\eta}$, which are extracted from the respective asymptotic behaviors, are identical. This proves our assertion a). In turn this shows that the difference of the phase shifts defined in eq. (3.16) is directly related to the resonance exchange. This proves our assertion b). In sect. 7 we will show that these assertions also hold when symmetry breaking is included even though that case is significantly more involved as, *e.g.*, $\lim_{N_C \rightarrow \infty} X_\Lambda \neq 0$.

6 The width of the Θ^+

In this section we will show that the Yukawa terms can be imposed to compute the width of the exchanged particles. For simplicity we will still omit flavor symmetry breaking.

6.1 Large- N_C limit

In a first step we show that in the large- N_C limit the iterated exchange, that is induced by the Yukawa terms in eq. (5.10), leads to the resonance phase shift discussed already in sect. 4. That is, we want to prove assertion b) by showing that the resonance width associated with $\delta_R(k)$ equals that induced by the Yukawa coupling of Θ^+ .

For large N_C we may omit the Λ exchange. Then the reaction matrix formalism is most suitable to compute $\delta_R(k)$ from the iterated exchange of a single resonance. The (real) reaction matrix obeys the Lippmann-Schwinger equation,

$$R = V + V \bar{G}_0 R, \quad (6.1)$$

where \bar{G}_0 and V are the Green's function for the constrained problem, eq. (3.15), and the separable potential due to the particle exchange, eq. (5.9), respectively. We stress that this is an exact solution to the scattering problem rather than any type of Born or distorted-wave Born approximation. The solution to the Lippmann-Schwinger equation is most conveniently obtained in a basis-independent formal language

$$\begin{aligned} \bar{G}_0(\omega) &= \frac{1}{\pi\omega} \mathcal{P} \int_0^\infty q^2 dq \left[\frac{|\bar{\eta}_{\omega_q}\rangle\langle\bar{\eta}_{\omega_q}|}{\omega - \omega_q} + \frac{|\bar{\eta}_{-\omega_q}\rangle\langle\bar{\eta}_{-\omega_q}|}{\omega + \omega_q} \right] \\ V(\omega) &= -\frac{\omega_0}{\omega_\Theta - \omega} |g_\Theta\rangle\langle g_\Theta| \quad \text{with} \\ \langle r | g_\Theta \rangle &= X_\Theta (2\lambda(r) - \omega_0 M_K(r)) z(r), \end{aligned} \quad (6.2)$$

where $\omega_p = \sqrt{p^2 + m_K^2}$ and \mathcal{P} denotes the principle value prescription. In the above expression $\bar{\eta}$ appears, rather than $\tilde{\eta}$, because the interaction Hamiltonian is treated as an additional interaction for the constrained fluctuations and we thus require the Green's function for $\bar{\eta}$. The potential is obtained from eq. (5.9) by setting $X_\Lambda = 0$. We find the reaction matrix

$$R = \frac{\omega_0 X_\Theta^2 | (2\lambda - \omega_0 M_K) z \rangle \langle z (2\lambda - \omega_0 M_K) |}{\omega - \omega_\Theta - \omega_0 \langle z (2\lambda) | \bar{G}_0(\omega) | (2\lambda) z \rangle} \quad (6.3)$$

since $\langle \bar{\eta}_\omega | M_K z \rangle = 0$. The resonance phase shift induced by V is obtained from the diagonal matrix element of the R -matrix, $k \langle \bar{\eta}_{\omega_k} | R | \bar{\eta}_{\omega_k} \rangle = -\tan(\delta_R(k))$, yielding

$$\tan(\delta_R(k)) = \frac{\Gamma(\omega_k)/2}{\omega_\Theta - \omega_k + \Delta(\omega_k)}. \quad (6.4)$$

This phase shift exhibits the canonical resonance structure with the width

$$\Gamma(\omega_k) = 2k\omega_0 X_\Theta^2 \left| \int_0^\infty r^2 dr z(r) 2\lambda(r) \bar{\eta}_{\omega_k}(r) \right|^2 \quad (6.5)$$

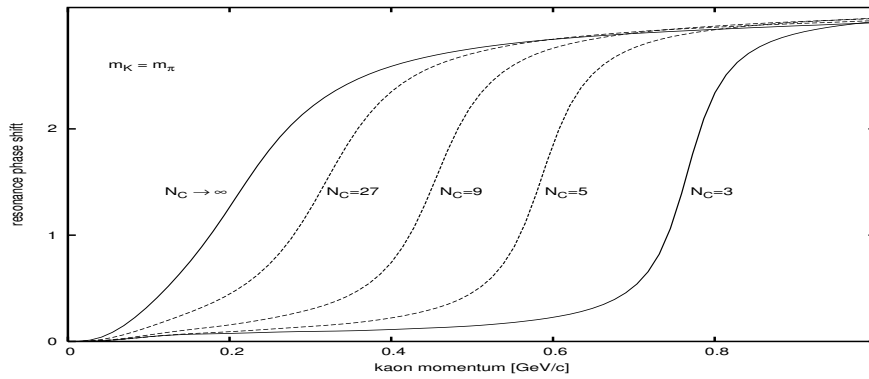


Fig. 5. The resonance phase shift as a function of N_C for $m_K = m_\pi$.

and the pole shift

$$\Delta(\omega_k) = \frac{1}{2\pi\omega_k} \mathcal{P} \int_0^\infty q dq \left[\frac{\Gamma(\omega_q)}{\omega_k - \omega_q} + \frac{\Gamma(-\omega_q)}{\omega_k + \omega_q} \right]. \quad (6.6)$$

Of course, we have numerically verified that in the large- N_C limit with $X_\Theta^2 = 1$, the phase shift from eq. (6.4) is identical to the one that we calculated as the difference in eq. (3.16). For finite N_C the width Γ acquires a factor $X_\Theta^2 \neq 1$ and the R -matrix formalism becomes two-dimensional (Λ and Θ^+ exchange).

We stress that although the singularity of the scattering amplitude is shifted by the amount Δ due to iterated resonance exchange, the “bare” mass of the resonance is ω_Θ , the pentaquark mass predicted in the RRA.

Alternatively we have applied Fermi’s golden rule to compute the transition rate $\Theta \rightarrow KN$ from the interaction Hamiltonian, eq. (5.5). This leads identically to the result displayed in eq. (6.5) and shows that the way we have included the separable potential in eq. (5.10) is unique.

6.2 Finite N_C

As already mentioned, the R -matrix formalism is two-dimensional at finite N_C because the Λ exchange cannot be omitted. It is therefore more convenient to compute the resonance phase shift in a fashion similar to eq. (3.16). First we compute the phase shift with both Yukawa interactions included from $\tilde{\eta}$ in eq. (5.10). From that we get the resonance phase shift by subtracting the background phase shift that we extract from $\tilde{\eta}$ in eq. (3.15). The resulting resonance phase shifts in the Θ^+ channel are displayed in fig. 5. It is illuminating to see how the BSA result emerges in the limit $N_C \rightarrow \infty$. Obviously this resonance becomes sharper and the pole shift decreases (in magnitude) to $\Delta = -14$ MeV as the number of color assumes its physical value $N_C = 3$. This is mainly due to the reduction of X_Θ . Furthermore, at finite N_C the bare resonance position is increased to $\omega_\Theta = (N_C + 3)/4\Theta_K$, according to eq. (4.14). This N_C -dependence of the resonance position is clearly exhibited in fig. 5.

There are two competing effects on the width when approaching the physical value, $N_C = 3$. The position

of the pole is increased by about a factor of two. This increases the available phase space and thus the width. On the other hand, the collective-coordinate matrix element X_Θ^2 decreases by approximately a factor 3, cf. eq. (5.9). According to eq. (6.5) this decreases the width.

At this point it is also illuminating to consider the large- N_C expansion of these matrix elements in more detail. For example,

$$X_\Theta^2 \rightarrow 1 - \frac{7}{N_C} + \frac{52}{N_C^2} - \frac{372}{N_C^3} + \frac{2628}{N_C^4} + \dots, \quad (6.7)$$

does not converge for $N_C \leq 7$. This clearly demonstrates that taking only leading pieces from a large- N_C expansion of these matrix elements yields incorrect results for $N_C = 3$. It is unavoidable to include finite N_C effects. Any approach [64] that employs $1/N_C$ expansion methods for exotic baryon matrix elements at $N_C = 3$ seems questionable.

6.3 Comparison with other approaches to compute the width

We stress that the appearance of only a single rather than two $SU(3)$ structures in the transition Hamiltonian in the flavor symmetric limit is not an artifact in the (eventually oversimplified) Skyrme model whose action is a functional of only pseudoscalar mesons. Rather it generalizes to *all* chiral soliton models. The reason is simple. Starting point in all these models is the introduction of intrinsic fluctuations about the classical soliton, *i.e.* the generalization of the *ansatz*, eq. (2.11) to parameterize additional fields. In this intrinsic formulation the interaction Hamiltonian must be an isoscalar operator that is linear in kaonic fluctuations and projects onto the P -wave channel. At $\mathcal{O}(N_C^0)$ only $d_{i\alpha\beta} \hat{x}_i \tilde{\eta}_\alpha \Omega_\beta$ is possible. Of course, in more extended models $\tilde{\eta}_\alpha$ also involves other strange degrees of freedom, *e.g.* K^* . In the flavor symmetric formulation the adjoint representation of the collective coordinates, D_{ab} , emerges *only* by the transformation to the laboratory system as in eq. (5.3). To see whether or not the limitation to only a single $SU(3)$ structure in the transition Hamiltonian is an artifact of the Skyrme model it is instructive to recall

Table 1. Matrix elements of collective-coordinate operators for $N_C = 3$ and $N_C \rightarrow \infty$ and various values of m_K , cf. eqs. (5.7), (7.3) and (8.6). The third entry, $m_K = 750$ MeV is chosen such as to compensate for the omitted derivative-type symmetry breaker.

	N_C	X_Θ	Y_Θ	X_Λ	Y_Λ	X_{Θ^*}	Y_{Θ^*}
$m_K = m_\pi$	3	0.596	0.149	-0.377	0.189	0.257	0.138
	∞	1	1	0	1	0.667	0.667
$m_K = 495$ MeV	3	0.690	0.168	-0.427	0.195	0.307	0.142
	∞	1.069	0.691	-0.378	0.691	0.713	0.461
$m_K = 750$ MeV	3	0.801	0.188	-0.497	0.203	0.376	0.147
	∞	1.157	0.575	-0.582	0.575	0.771	0.383

how the authors of refs. [10,41] came to the improper use of several $SU(3)$ structures regardless of flavor symmetry breaking. Starting point of those studies is the spatial part of the axial current and its identification with the kaon field operator via PCAC. This axial current is commonly computed from the rigidly rotating classical field only and in general allows for three $SU(3)$ structures in the flavor symmetric case. One of those structures is directly related to the presumably small singlet current matrix element and is usually ignored in the context of the Θ^+ width. In the simple Skyrme model it vanishes exactly. However, the other two structures are also present in the Skyrme model axial current [65]. They simply do not show up in the transition Hamiltonian. Stated otherwise, the axial current operator is not suitable to compute hadronic-decay widths in chiral soliton models. A simple reason is that the isospin embedding of the hedgehog soliton, eq. (2.8), breaks flavor symmetry. In particular this prevents using $SU(3)$ to relate various hadron decay widths among each other. For example, the computation of the $\Delta \rightarrow \pi N$ width requires a term linear in the intrinsic pion fluctuations which involves collective-coordinate operators that are not related to $d_{i\alpha\beta}\hat{x}_i\tilde{\eta}_\alpha\Omega_\beta$.

Of course, in our approach $\partial^\mu A_\mu^a = f_\pi m_a^2 \phi^a + \mathcal{O}(\phi^3)$, *i.e.* PCAC, holds as well. It is nothing but the equation of motion for the chiral angle and the meson fluctuations that we solve. That is, in our approach PCAC is satisfied at $\mathcal{O}(N_C)$ and $\mathcal{O}(N_C^0)$ while in refs. [10,41] the equation of motion is only considered at $\mathcal{O}(N_C)$. The main point, however, is that within soliton models the right-hand side may not be the interpolating field for the pseudoscalar meson in the final state of hadronic decays.

7 $SU(3)$ symmetry breaking

In sect. 5 we have established the equivalence of the bound-state and rotation-vibration coupling approaches for computing the kaon nucleon phase shift in the large- N_C limit in the flavor symmetric case. Here we will extend that discussion to the symmetry breaking case.

The interaction Hamiltonian that describes the Yukawa exchange for fluctuations in the laboratory frame, cf. eq. (5.5), acquires an additional term from the symme-

try breaking part of the action, I_{SB} ,

$$\begin{aligned}
H_{\text{int}} = & \frac{2f_\pi}{\Theta_K} d_{i\alpha\beta} D_{\gamma\alpha} R_\beta \int d^3r \sin \frac{F(r)}{2} \\
& \times [2\lambda(r) - \omega_0 M_K(r)] \tilde{\xi}_\gamma(\vec{x}, t) \hat{x}_i \\
& - \frac{4f_\pi}{\sqrt{3}} d_{i\alpha\beta} D_{\gamma\alpha} D_{8\beta} \int d^3r \sin \frac{F(r)}{2} \\
& \times \left[m_K^2 - m_\pi^2 - \frac{3\Gamma}{8\Theta_K} M_K(r) \right] \tilde{\xi}_\gamma(\vec{x}, t) \hat{x}_i. \quad (7.1)
\end{aligned}$$

We provide the detailed derivation of this interaction Hamiltonian in the appendix. In principle, there are many pole contributions, such as from states with $S = \pm 1$ and $I = 0$ that stem from higher-dimensional $SU(3)$ representations. They lead to sharp resonances at higher energies and may be omitted for the energy regime of current interest. We thus keep only the contributions associated with the exchanges of Λ and Θ^+ . Then the separable Yukawa exchange potential in the P -wave has the general form

$$\begin{aligned}
V(\omega) = & -\frac{\omega_0}{\omega_\Theta - \omega} |g_\Theta\rangle \langle g_\Theta| - \frac{\omega_0}{\omega_\Lambda + \omega} |g_\Lambda\rangle \langle g_\Lambda| \\
\langle r | g_\nu \rangle = & X_\nu (2\lambda(r) - \omega_0 M_K(r)) z(r) \\
& + \frac{Y_\nu}{\omega_0} \left(m_K^2 - m_\pi^2 - \frac{3\Gamma}{8\Theta_K} M_K(r) \right) z(r), \quad \nu = \Theta, \Lambda. \quad (7.2)
\end{aligned}$$

Note that $\langle z | g_\nu \rangle = 0$. This implies that the solutions to the constrained equation of motion (3.15) with this potential added, also do satisfy the constraint $\langle z | M_K \tilde{\eta} \rangle = 0$.

The matrix elements X_ν have already been defined in eq. (5.7). Similarly we introduce

$$\begin{aligned}
\langle \Theta^+ | d_{3\alpha\beta} D_{+\alpha} D_{8\beta} | n \rangle = & Y_\Theta \sqrt{\frac{3}{8N_C}}, \\
\langle \Lambda | d_{3\alpha\beta} D_{-\alpha} D_{8\beta} | p \rangle = & Y_\Lambda \sqrt{\frac{3}{8N_C}}, \quad (7.3)
\end{aligned}$$

where Y_Θ and Y_Λ tend to unity in the flavor symmetric case as $N_C \rightarrow \infty$. Of course, for non-zero symmetry breaking the matrix elements X_ν and Y_ν have to be computed numerically and cannot be given as simple functions of N_C , results are listed in table 1.

The integro-differential equation with the Yukawa pieces included is a bit more complicated than in the flavor

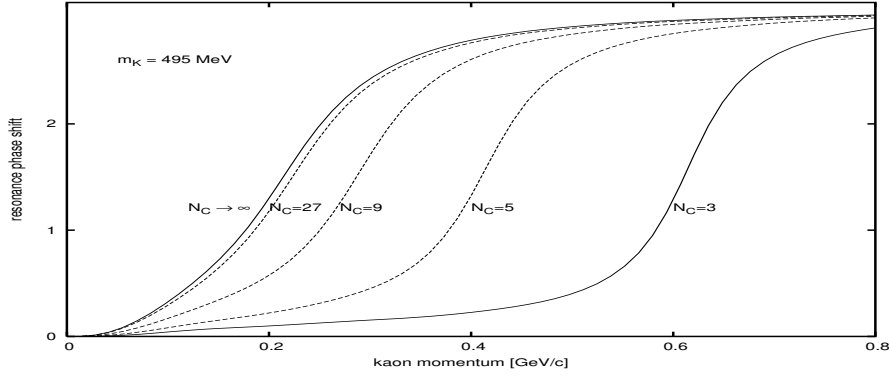


Fig. 6. The resonance phase shift as a function of N_C for $m_K = 495$ MeV.

symmetric case, eq. (5.10),

$$\begin{aligned}
 h^2 \tilde{\eta} + \omega (2\lambda - \omega M_K) \tilde{\eta} = & - \left[2\lambda - (\omega + \omega_0) M_K \right. \\
 & - \omega_0 \left(\frac{X_\Theta^2}{\omega_\Theta - \omega} + \frac{X_\Lambda^2}{\omega_\Lambda + \omega} \right) (2\lambda - \omega_0 M_K) \\
 & \left. - \left(\frac{X_\Theta Y_\Theta}{\omega_\Theta - \omega} + \frac{X_\Lambda Y_\Lambda}{\omega_\Lambda + \omega} \right) \left(m_K^2 - m_\pi^2 - \frac{3\Gamma}{8\Theta_K} M_K \right) \right] \\
 & \times z \int_0^\infty dr r^2 z (2\lambda) \tilde{\eta} + (m_K^2 - m_\pi^2) \\
 & \times \left[M_K + \left(\frac{X_\Theta Y_\Theta}{\omega_\Theta - \omega} + \frac{X_\Lambda Y_\Lambda}{\omega_\Lambda + \omega} \right) (2\lambda - \omega_0 M_K) \right. \\
 & \left. + \frac{1}{\omega_0} \left(\frac{Y_\Theta^2}{\omega_\Theta - \omega} + \frac{Y_\Lambda^2}{\omega_\Lambda + \omega} \right) \left(m_K^2 - m_\pi^2 - \frac{3\Gamma}{8\Theta_K} M_K \right) \right] \\
 & \times z \int_0^\infty dr r^2 z \tilde{\eta}. \tag{7.4}
 \end{aligned}$$

For simplicity we have omitted the arguments of the radial functions.

Let $\eta(r)$ again be a solution to the BSA equation (3.5). In the limit $N_C \rightarrow \infty$ we require that $\tilde{\eta} = \eta - az$ with $a = \int_0^\infty dr r^2 z(r) M_K(r) \eta(r)$ is a solution to eq. (7.4) also in the case that symmetry breaking ($m_K \neq m_\pi$) is included. Modulo phase conventions this implies

$$X_\Theta = \frac{\omega_\Theta}{\omega_0} Y_\Theta, \quad X_\Lambda = -\frac{\omega_\Lambda}{\omega_0} Y_\Lambda, \quad Y_\Theta = Y_\Lambda = \sqrt{\frac{\omega_0}{\omega_\Theta + \omega_\Lambda}}, \tag{7.5}$$

as $N_C \rightarrow \infty$. And indeed, the matrix elements in eqs. (5.7) and (7.3) give these results when computed with the *exact* solutions to the eigenvalue equation (4.15) with flavor symmetry breaking included! In table 1 we display numerical results for these matrix elements for the physical case $N_C = 3$ and the large- N_C limit for various values of m_K . The third entry, $m_K = 750$ MeV yields the value of the symmetry breaking parameter Γ that we would have obtained if we had included symmetry breaking terms that contain derivatives of the chiral field. Obviously, there are significant deviations from the large- N_C limit at $N_C = 3$.

The large- N_C equation of motion for $\tilde{\eta}$ has a bound-state solution in the antikaon channel exactly at the BSA energy $\omega_\Lambda = 196$ MeV (for $m_K = 495$ MeV). This is

a starting point to discuss corrections to the RRA spectrum caused by the rotation-vibration coupling for finite N_C which improves on earlier attempts that introduced fluctuations induced by the rigid rotations [47]. However, that discussion is beyond the scope of this paper.

8 Soliton model predictions for Θ^+

We have already noted that in the flavor symmetric case the excitation energy of Θ^+ is altered according to eq. (4.14) for finite N_C . The excitation energy of Λ remains at zero. For non-zero symmetry breaking the situation is more complicated as both excitation energies depend on the value for m_K . That is, we need to substitute

$$\omega_\Lambda \longrightarrow E_\Lambda - E_N \quad \text{and} \quad \omega_\Theta \longrightarrow E_\Theta - E_N, \tag{8.1}$$

where $E_{N,\Theta,\Lambda}$ are the solutions to the eigenvalue problem, eq. (4.15) in the respective channel for a prescribed $N_C < \infty$. In addition, we employ the corresponding eigenstates of eq. (4.15) to evaluate the matrix elements in eqs. (5.7) and (7.3) to numerically compute $X_\Theta, X_\Lambda, Y_\Theta$ and Y_Λ at any value of N_C and m_K , cf. table 1. We then compute the resonance phase shift as described in sect. 6.2. The results are shown in fig. 6. Again these phase shifts exhibit a pronounced resonance structure. As in the flavor symmetric case, finite- N_C effects cause the pole position to move to higher momenta and also the resonance to become sharper. Although the phase shifts displayed in figs. 5 and 6 as a function of momentum seem to be similar, this is not the case for the resonance energy since it is subject to the dispersion relation $\omega = \sqrt{k^2 + m_K^2}$ and m_K differs significantly in the two cases. From fig. 6 we read off a Θ -nucleon mass difference of about 750 MeV which is a bit higher than the empirical value of about 600 MeV if the Θ^+ signal around 1540 MeV were indeed established. We remark, however, that the actual prediction for this mass difference is quite model dependent, *e.g.* we have omitted effects due to $f_K \neq f_\pi$. Also, the inclusion of other fields like scalar or vector mesons or chiral quarks will alter the quantitative results.

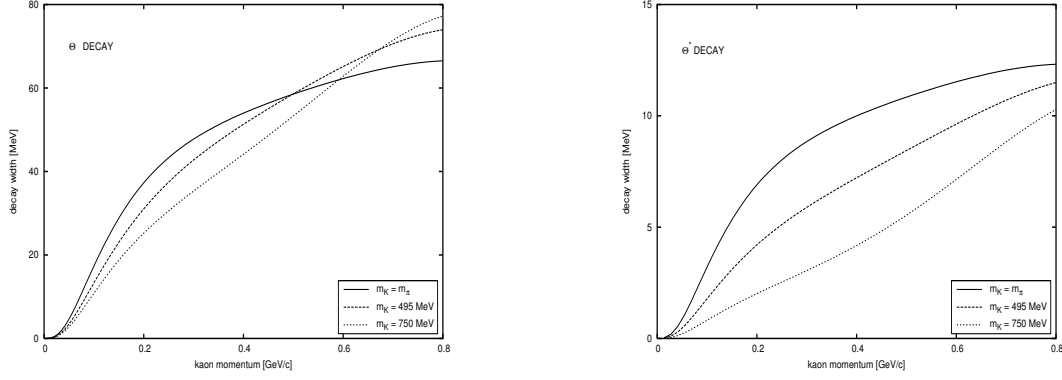


Fig. 7. Model prediction for the width, $\Gamma(\omega)$, of Θ^+ (left) and Θ^* (right) for $N_C = 3$ as a function of the momentum $k = \sqrt{\omega^2 - m_K^2}$ for three values of the kaon mass. Note that the two panels have different scales.

The width function, $\Gamma(\omega)$ is computed from generalizing eq. (6.5) to also contain symmetry breaking effects,

$$\Gamma(\omega_k) = 2k\omega_0 \left| X_\Theta \int_0^\infty r^2 dr z(r) 2\lambda(r) \bar{\eta}_{\omega_k}(r) + \frac{Y_\Theta}{\omega_0} (m_K^2 - m_\pi^2) \int_0^\infty r^2 dr z(r) \bar{\eta}_{\omega_k}(r) \right|^2, \quad (8.2)$$

which arise from the Θ^+ matrix elements of the separable potential, eq. (7.2). Again, $\bar{\eta}$ rather than $\tilde{\eta}$ appears in the above expression. Note that contributions of the form $\int dr r^2 z(r) M_K(r) \bar{\eta}(r)$ vanish due to the second constraint in eq. (3.12).

We have already mentioned that chiral soliton models only provide qualitative insight into the baryon mass spectrum. At best, we expect the model predictions for masses of exotic baryons to be reliable at the, say, 100 MeV level. Let us therefore now assume that the Θ^+ -resonance indeed corresponds to the recently asserted pentaquark of mass $M_\Theta = 1540$ MeV and estimate its width from $\Gamma(\omega)$. First, we have to find the corresponding kaon momentum. This is not without ambiguity because in the soliton picture baryon masses are of higher order in N_C than the meson fluctuations and thus considered infinitely heavy. Therefore, recoil effects are not necessarily included. This yields $k = 340$ MeV for the kaon momentum. Of course, a simple calculation using relativistic kinematics incorporates recoil effects and leads to $k = 270$ MeV. For these two momenta we respectively read off widths of 47 MeV and 40 MeV for $m_K = 495$ MeV from fig. 7. Fortunately the dependence of $\Gamma(\omega)$ on the effective strength of symmetry breaking seems only moderate in the resonance region. Hence the extracted width is not too sensitive on the uncertainties stemming from the omission of the derivative-type symmetry breaker and we finally estimate $\Gamma \approx 40$ MeV as the width of the Θ^+ pentaquark. Though we do not expect chiral soliton model predictions for the widths to be reliable at the 10 MeV level, we note that this result should be considered a small number as it has to be compared to widths of other hadronic decays of baryon resonances, *e.g.* $\Gamma(\Delta \rightarrow \pi N) \approx 120$ MeV, empirically.

Finally let us briefly comment on the width of the Θ^* pentaquark, the $J = \frac{3}{2}$ and $I = 1$ partner of Θ^+ . In large

N_C the starting point is again the intrinsic T -matrix of the BSA. However, for Θ^* the recoupling between intrinsic laboratory frames is more complicated than eq. (3.1). There are two intrinsic T -matrix elements $T_{\mathcal{G}l} = T_{\frac{1}{2},1}$ and $T_{\frac{3}{2},1}$ with different grand spins which contribute to the scattering in the laboratory frame [26, 52],

$$T(P13) = \frac{4}{9} T_{\frac{1}{2},1} + \frac{5}{9} T_{\frac{3}{2},1}. \quad (8.3)$$

While the collective excitation and thus the relevant phase shift displayed in figs. 1 and 2 emerge in the intrinsic $\mathcal{G} = \frac{1}{2}$ channel, the $\mathcal{G} = \frac{3}{2}$ contributes only to the background phase shift and is irrelevant for the Θ^* -resonance structure. From eq. (8.3) we notice a factor $\frac{4}{9}$ not present in the corresponding relation for Θ^+ , eq. (3.1).

In the notation of the RRA and for $m_K = m_\pi$ the Θ^* dwells in the “27”-plet of flavor $SU(3)$ for finite N_C , while for $N_C \rightarrow \infty$ it becomes degenerate with Θ^+ . For $N_C = 3$ the Θ^* pentaquark is expected to be roughly 100 MeV heavier than Θ^+ [39, 59]. The Yukawa interactions of the RVA, eq. (7.1) now lead to (iterated) Θ^* and Σ^* exchanges.

The matrix elements relevant to compute the width of Θ^* are reduced via

$$\begin{aligned} \sum_{i=\pm\frac{1}{2}} \langle \Theta^* | a_{1,m}^{(i)} d_{m\alpha\beta} D_{\gamma(i)\alpha} R_\beta | (KN)_{I=1, J=\frac{3}{2}} \rangle &= \\ &= \sqrt{3} \langle \Theta^{*+} \uparrow | d_{3\alpha\beta} D_{+\alpha} R_\beta | n \uparrow \rangle, \\ \sum_{i=\pm\frac{1}{2}} \langle \Theta^* | a_{1,m}^{(i)} d_{m\alpha\beta} D_{\gamma(i)\alpha} D_{8\beta} | (KN)_{I=1, J=\frac{3}{2}} \rangle &= \\ &= \sqrt{3} \langle \Theta^{*+} \uparrow | d_{3\alpha\beta} D_{+\alpha} D_{8\beta} | n \uparrow \rangle. \end{aligned} \quad (8.4)$$

The relative factor $1/\sqrt{2}$ in comparison with eq. (5.6) is essential so that for Θ^* we have to replace $X_\Theta \rightarrow X_{\Theta^*}$ and $Y_\Theta \rightarrow Y_{\Theta^*}$ in eq. (8.2) by

$$\begin{aligned} X_{\Theta^*} &= \frac{4}{\sqrt{N_C}} \langle \Theta^{*+} \uparrow | d_{3\alpha\beta} D_{+\alpha} R_\beta | n \uparrow \rangle \quad \text{and} \\ Y_{\Theta^*} &= 2\sqrt{\frac{N_C}{3}} \langle \Theta^{*+} \uparrow | d_{3\alpha\beta} D_{+\alpha} D_{8\beta} | n \uparrow \rangle. \end{aligned} \quad (8.5)$$

In the flavor symmetric case we find

$$X_{\Theta^*} = \frac{2}{3}(N_C - 1) \sqrt{\frac{N_C + 5}{N_C(N_C + 3)(N_C + 9)}} = \frac{2}{3} \begin{cases} \frac{2}{9}\sqrt{3}, & N_C = 3, \\ 1, & N_C = \infty. \end{cases} \quad (8.6)$$

Thus, disregarding phase space effects (due to Θ^* lying roughly 100 MeV above Θ^+ for $N_C = 3$), we obtain

$$\Gamma_{\Theta^*} = \Gamma_{\Theta} \left(\frac{X_{\Theta^*}}{X_{\Theta}} \right)^2 = \Gamma_{\Theta} \begin{cases} \frac{5}{27}, & N_C = 3, \\ \frac{4}{9}, & N_C = \infty. \end{cases} \quad (8.7)$$

From eq. (8.6) we notice that in the large- N_C limit the factor $\frac{4}{9}$ that appears in eq. (8.3) is recovered. This is precisely what is required in order to obtain the BSA result in this limit. We have numerically verified that in the symmetry breaking case

$$\lim_{N_C \rightarrow \infty} \frac{X_{\Theta^*}}{X_{\Theta}} = \lim_{N_C \rightarrow \infty} \frac{Y_{\Theta^*}}{Y_{\Theta}} = \frac{2}{3} \quad (8.8)$$

for any value $m_K \geq m_{\pi}$ such that $\Gamma_{\Theta^*} = \frac{4}{9}\Gamma_{\Theta}$ in that case as well, see also table 1. Furthermore, from eq. (8.7) we also notice that for $N_C = 3$ the Θ^* width is even more suppressed in comparison to Θ^+ (by roughly a factor of five in the symmetric case.) Finite symmetry breaking lowers the prediction for the Θ^* width even further, cf. fig. 7. Thus, we expect a very sharp Θ^* -resonance with a width of roughly 10 MeV about 100 MeV above the Θ^+ pentaquark. This is opposite to the above-criticized scenario based on matrix elements of the axial current to compute widths for hadronic decays, *e.g.* the authors of ref. [41] obtained $\Gamma_{\Theta^*}/\Gamma_{\Theta} = 3, \dots, 6$.

9 Conclusions

We have thoroughly compared the bound-state (BSA) and the collective-coordinate approaches to chiral soliton models in flavor $SU(3)$. For definiteness we have only considered the simplest version of the Skyrme model augmented by the Wess-Zumino and symmetry breaking terms. However, our analysis merely concerns the quantization of fluctuations about the classical soliton that generates baryons states. Therefore our qualitative results are valid for *any* chiral soliton model.

Often the collective-coordinate approach is identified with the rigid-rotator approach (RRA) as only quantizing the spin-flavor orientation of the soliton. But, a sensible comparison with the BSA requires the inclusion of fluctuations in addition to the collective coordinates. This generalizes to the rotation-vibration approach (RVA). Here we have studied that scenario exhaustively and in particular ensured that the introduction of such fluctuations does not double-count any degrees of freedom. Only in this way the model generates an interaction Hamiltonian that describes hadronic decays. In doing so, we have solved

the long-standing Yukawa problem in the kaon sector. Technically the derivation of this Hamiltonian is quite involved; however, the result is as simple as convincing: In the limit $N_C \rightarrow \infty$, in which the BSA is undoubtedly correct, the RVA and BSA yield identical results for both the baryon spectrum as well as the kaon-nucleon S -matrix. This equivalence also holds when flavor symmetry breaking is included. In the first place this ensures that we have correctly introduced the collective coordinates. On top of that, this result is very encouraging as it clearly demonstrates that collective-coordinate quantization is valid regardless of whether or not the respective modes are zero-modes. Though the large- N_C limit is helpful for testing the results of the RVA, we have also seen that taking only leading terms in the respective matrix elements is not trustworthy.

There has been quite some confusion from the potential disagreements between the BSA and the RVA. There are essentially two reasons for that confusion:

- 1) The BSA yields a slowly rising phase shift in the $S = +1$ channel that led to the misinterpretation that no resonance was present. However, the scenario is that there is indeed a (broad) resonance which, unfortunately, is hidden by a repulsive background.
- 2) The Θ^+ width was incorrectly computed. Those computations did not even attempt to make contact with the BSA.

Here we have resolved these puzzles: We observe a relatively sharp resonance in the RVA for $N_C = 3$ that evolves into the broad structure seen in the BSA as N_C increases.

Another major result of our calculation is that in the flavor symmetric case the interaction Hamiltonian contains only a single leading N_C structure of $SU(3)$ matrix elements for the $\Theta^+ \rightarrow KN$ transition. Any additional structure only enters via flavor symmetry breaking terms in the Lagrangian. This is at odds with earlier approaches that essentially assumed any possible structure and fitted coefficients from a variety of hadronic decays under the assumption of $SU(3)$ relations. We stress that this is not a valid procedure as already the embedding of the classical soliton breaks $SU(3)$ and thus yields different structures for different hadronic transitions. In particular, strangeness-conserving and strangeness-changing processes are not related to each other in chiral soliton model treatments. As for $m_K = m_{\pi}$ only a single transition operator exists, there cannot occur any cancellation between different (leading N_C) structures to explain the probably small Θ^+ width.

Though these general results are model independent, the numerical results that we obtain for the masses and the widths of pentaquarks are not. Moreover, the prediction for the latter may suffer from those subleading $1/N_C$ contributions that were not considered here. Our numerical estimates indicate that the Θ^+ is roughly 700 MeV heavier than the nucleon with quite a small width of about 40 MeV. We predict the Θ^* to be about 100 MeV heavier than the Θ^+ with an even smaller width of the order of 10 MeV. It should be kept in mind, that all quantitative results for masses and widths are model dependent and

that their accurate predictions are very delicate. Despite that, chiral soliton models do predict the existence of the low-lying pentaquarks Θ and Θ^* with strangeness, $S = +1$ and isospin, $I = 0, 1$. This is irrespective of details of the model and/or the adopted approach.

Let us finally speculate about implications that indications for pentaquarks seen earlier in kaon nucleon scattering [66,67] may have on chiral soliton models. At that time the structures observed in the $S = +1$ channel were assigned the quantum numbers $P_{01}(1830)$, $P_{13}(1810)$, $D_{03}(1790)$ and $D_{15}(2070)$. In chiral soliton models the latter can only be interpreted as a quadrupole excitation of a rotational ground state in the P_{01} channel. The structure observed in that channel is actually higher in energy than both the P_{13} and D_{03} structures. It is thus suggestive that this P_{01} structure should not be identified with the rotational ground state whose mass should thus be significantly lower than 1790 MeV. This would be a further hint for a Θ^+ pentaquark with $M_\Theta \lesssim 1700$ MeV.

We very much appreciate interesting and helpful discussions with G. Holzwarth and V. Kopeliovich.

Additional remark. After this manuscript had been submitted, it has been commented on by T. Cohen [68]. We have rebutted his arguments in a reply [69].

Appendix A. Constrained equations of motion

In this appendix we will derive the equation of motion (3.15) for the fluctuations in the RVA. We find it more convenient to perform that analysis in a general language for $\tilde{\eta}_\alpha$ rather than for the P -wave projection, eq. (3.2). In that language the kaonic zero-mode wave function carries two indices. They parameterize the rotation of kaon degrees of freedom ($\alpha = 4, \dots, 7$) into any direction ($a = 1, \dots, 8$) of $SU(3)$,

$$z_\alpha^a(\vec{x}) = \frac{2f_\pi}{\sqrt{\Theta_K}} \sin \frac{F(r)}{2} \hat{x}_i f_{i\alpha a}. \quad (\text{A.1})$$

Only four of these wave functions ($b = \beta = 4, \dots, 7$) are non-zero. They correspond to different choices of the zero-mode analogue of the complex isospinor in eq. (3.2) and are normalized such that $\int d^3r z_\alpha^\beta(\vec{x}) M_K(r) z_\alpha^\gamma(\vec{x}) = \delta_{\beta\gamma}$, where $M_K(r)$ is the metric function defined in eq. (3.4). The analogue of eq. (3.7) reads

$$\int d^3r 2\lambda(r) f_{8\alpha\beta} z_\alpha^\gamma(\vec{x}) z_\beta^\delta(\vec{x}) = \omega_0 f_{8\gamma\delta}. \quad (\text{A.2})$$

We postpone the discussion of the rotation-vibration coupling terms involving the angular velocity Ω_8 to the end of this appendix and collect all other terms in the Lagrange function that are $\mathcal{O}(N_C^0)$ according to the rules, eq. (5.1).

The result of this tedious calculation is

$$\begin{aligned} L = & -M - \frac{N_C}{2\sqrt{3}} \Omega_8 + \frac{1}{2} \Theta_K \Omega_\alpha^2 \\ & + \frac{1}{2} \int d^3r \left[M_K(r) \dot{\tilde{\eta}}_\alpha(\vec{x}) \dot{\tilde{\eta}}_\alpha(\vec{x}) - \tilde{\eta}_\alpha(\vec{x}) h_{\alpha\beta}^2(\vec{x}) \tilde{\eta}_\beta(\vec{x}) \right] \\ & + \frac{2}{\sqrt{3}} \int d^3r \lambda(r) f_{8\alpha\beta} \tilde{\eta}_\alpha(\vec{x}) \dot{\tilde{\eta}}_\beta(\vec{x}) - \sqrt{\Theta_K} \Omega_\alpha \\ & \times \int d^3r z_\beta^\alpha(\vec{x}) \left[M_K(r) \dot{\tilde{\eta}}_\beta(\vec{x}) - \frac{4}{\sqrt{3}} \lambda(r) f_{8\beta\gamma} \tilde{\eta}_\gamma(\vec{x}) \right] \\ & + \frac{4}{3} \sqrt{\Theta_K} (m_K^2 - m_\pi^2) f_{8\alpha\beta} D_{8\alpha} \\ & \times \int d^3r z_\gamma^\beta(\vec{x}) \tilde{\eta}_\gamma(\vec{x}) + \mathcal{O} \left(\frac{1}{\sqrt{N_C}} \right), \end{aligned} \quad (\text{A.3})$$

where dots denote derivatives with respect to time. Since we are mainly interested in relations between conjugate momenta, the explicit form of the Hermitian potential $h_{\alpha\beta}^2(\vec{x})$ (that also contains the spatial derivative operators as indicated in eq. (3.4)) is irrelevant. For the Skyrme model Lagrangian, eq. (2.3), it may be traced from the literature, *e.g.* refs. [29,54]. The last two terms in eq. (A.3) are new and describe the coupling between fluctuations and the collective coordinates. The very last term would be absent in a flavor symmetric world. In that case only a single $SU(3)$ operator, that is linear in the kaonic angular velocity Ω_α , couples the fluctuations to the collective coordinates. We will soon see that in the contribution proportional to Ω_α in eq. (A.3) only the piece that stems from the Wess-Zumino term survives because of the constraints.

The conjugate momenta of the kaon degrees of freedom are

$$\begin{aligned} R_\alpha = & -\frac{\partial L}{\partial \Omega_\alpha} = -\Theta_K \Omega_\alpha + \sqrt{\Theta_K} \int d^3r z_\beta^\alpha(\vec{x}) \\ & \times \left[M_K(r) \dot{\tilde{\eta}}_\beta(\vec{x}) - \frac{4}{\sqrt{3}} \lambda(r) f_{8\beta\gamma} \tilde{\eta}_\gamma(\vec{x}) \right], \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \Pi_\alpha(\vec{x}) = & \frac{\delta L}{\delta \tilde{\eta}_\alpha(\vec{x})} = -\sqrt{\Theta_K} \Omega_\beta M_K(r) z_\alpha^\beta(\vec{x}) \\ & + M_K(r) \dot{\tilde{\eta}}_\alpha(\vec{x}) - \frac{2}{\sqrt{3}} \lambda(r) f_{8\beta\gamma} \tilde{\eta}_\gamma(\vec{x}). \end{aligned} \quad (\text{A.5})$$

Obviously, they are linearly dependent,

$$R_\alpha = \sqrt{\Theta_K} \int d^3r z_\beta^\alpha(\vec{x}) \left[\Pi_\beta(\vec{x}) - \frac{2}{\sqrt{3}} \lambda(r) f_{8\beta\gamma} \tilde{\eta}_\gamma(\vec{x}) \right]. \quad (\text{A.6})$$

The second piece stems from the Wess-Zumino term. It is unexpected because the conjugate momenta originate from parts in the Lagrangian that contain time derivatives of the chiral field, U . In turn the linear dependence between the conjugate momenta results from the connection between $\frac{\partial \tilde{U}(\vec{x}, t)}{\partial \Omega_\alpha}$ and $\frac{\delta \tilde{U}(\vec{x}, t)}{\delta \tilde{\eta}_\alpha(\vec{x})}$ causing a linear dependence between R_α and Π_α only. However, this argument only holds for *local* pieces of the action while the Wess-Zumino term is *non-local*.

We disentangle the collective and fluctuation pieces in eq. (A.5)

$$\begin{aligned} \Pi_\alpha^{(\text{coll})}(\vec{x}) &= -\sqrt{\Theta_K} \Omega_\beta M_K(r) z_\alpha^\beta(\vec{x}) \quad \text{and} \\ \tilde{\Pi}_\alpha(\vec{x}) &= M_K(r) \dot{\tilde{\eta}}_\alpha(\vec{x}) - \frac{2}{\sqrt{3}} \lambda(r) f_{8\alpha\beta} \tilde{\eta}_\beta(\vec{x}) \end{aligned} \quad (\text{A.7})$$

and demand that the momenta conjugate to the collective coordinates do not contain any fluctuation parts,

$$R_\alpha = \sqrt{\Theta_K} \int d^3r z_\beta^\alpha(\vec{x}) \Pi_\beta^{(\text{coll})}(\vec{x}) = -\Theta_K \Omega_\alpha. \quad (\text{A.8})$$

The linear relation eq. (A.6) then translates into the *primary constraints*,

$$\Psi_\alpha = \int d^3r z_\beta^\alpha(\vec{x}) \left[\tilde{\Pi}_\beta(\vec{x}) - \frac{2}{\sqrt{3}} \lambda(r) f_{8\beta\gamma} \tilde{\eta}_\gamma(\vec{x}) \right] = 0. \quad (\text{A.9})$$

The corresponding *secondary constraints* require the fluctuations to be orthogonal to the zero-mode,

$$\chi_\alpha = \int d^3r z_\beta^\alpha(\vec{x}) M_K(r) \tilde{\eta}_\beta(\vec{x}) = 0. \quad (\text{A.10})$$

These constraints are linear functionals of the fluctuations and their conjugate momenta. They satisfy the Poisson brackets $\{\Psi_\alpha, \chi_\beta\} = \delta_{ab}$ so that $\tilde{\Pi}_\alpha$ and $\tilde{\eta}_\beta$ are conjugate to each other in the constrained subspace [70]. This is unaltered by the (unexpected) contribution to Ψ_α from the Wess-Zumino term. After projection onto the *P*-wave channel, the constraints eqs. (A.9) and (A.10) turn into constraints for the associated radial functions given in eq. (3.12).

We need to find the equations of motion for $\tilde{\Pi}_\alpha$ and $\tilde{\eta}_\alpha$ and also to extract the Hamiltonian for the interaction between the collective and fluctuation degrees of freedom. We start by the Legendre transformation and add the constraints Ψ_α and χ_α with Lagrange multipliers α_α and β_α , respectively,

$$H = -L - R_\alpha \Omega_\alpha + \int d^3r \Pi_\alpha(\vec{x}) \dot{\tilde{\eta}}_\alpha(\vec{x}) + \alpha_\alpha \Psi_\alpha + \beta_\alpha \chi_\alpha. \quad (\text{A.11})$$

We express this $\mathcal{O}(N_C^0)$ Hamiltonian in terms of the conjugate fields $\tilde{\Pi}_\alpha$ and $\tilde{\eta}_\alpha$ as well as the angular momenta R_α ,

$$\begin{aligned} H &= \frac{R_\alpha^2}{2\Theta_K} + \frac{1}{2} \int \frac{d^3r}{M_K(r)} \left[\tilde{\Pi}_\alpha(\vec{x}) + \frac{2}{\sqrt{3}} \lambda(r) f_{8\alpha\beta} \tilde{\eta}_\beta(\vec{x}) \right]^2 \\ &\quad + \frac{1}{2} \int d^3r \tilde{\eta}_\alpha(\vec{x}) h_{\alpha\beta}^2(\vec{x}) \tilde{\eta}_\beta(\vec{x}) + \frac{R_\alpha}{\sqrt{\Theta_K}} \\ &\quad \times \int d^3r z_\beta^\alpha(\vec{x}) \left[\tilde{\Pi}_\alpha(\vec{x}) + \frac{2}{\sqrt{3}} \lambda(r) f_{8\alpha\beta} \tilde{\eta}_\beta(\vec{x}) \right] \\ &\quad + \alpha_\alpha \Psi_\alpha - \frac{4}{3} \sqrt{\Theta_K} (m_K^2 - m_\pi^2) f_{8\alpha\beta} D_{8\alpha} \\ &\quad \times \int d^3r z_\gamma^\beta(\vec{x}) \tilde{\eta}_\gamma(\vec{x}) + \beta_\alpha \chi_\alpha. \end{aligned} \quad (\text{A.12})$$

The Hamiltonian obviously contains terms that are explicitly linear in the fluctuations or their conjugate momenta. These terms contribute to the Yukawa interaction, eq. (7.1). Additional linear terms arise from the Lagrange multipliers which we compute in two steps from the equations of motion,

$$\begin{aligned} \dot{\tilde{\eta}}_\alpha(\vec{x}) &= \frac{\delta H}{\delta \tilde{\Pi}_\alpha(\vec{x})} = \frac{1}{M_K(r)} \left[\tilde{\Pi}_\alpha(\vec{x}) + \frac{2}{\sqrt{3}} \lambda(r) f_{8\alpha\beta} \tilde{\eta}_\beta(\vec{x}) \right] \\ &\quad + \left(\frac{R_\beta}{\sqrt{\Theta_K}} + \alpha_\alpha \right) z_\alpha^\beta(\vec{x}), \\ \dot{\tilde{\Pi}}_\alpha(\vec{x}) &= \frac{\delta H}{\delta \tilde{\eta}_\alpha(\vec{x})} = -h_{\alpha\beta}^2(\vec{x}) \tilde{\eta}_\beta(\vec{x}) \\ &\quad + \frac{2}{\sqrt{3}} \frac{\lambda(r)}{M_K(r)} f_{8\alpha\beta} \left[\tilde{\Pi}_\alpha(\vec{x}) + \frac{2}{\sqrt{3}} \lambda(r) f_{8\alpha\beta} \tilde{\eta}_\beta(\vec{x}) \right] \\ &\quad + \frac{2}{\sqrt{3}} \left(\frac{R_\beta}{\sqrt{\Theta_K}} - \alpha_\alpha \right) \lambda(r) f_{8\alpha\gamma} z_\gamma^\beta(\vec{x}) \\ &\quad + \frac{4}{3} \sqrt{\Theta_K} (m_K^2 - m_\pi^2) f_{8\beta\gamma} D_{8\beta} z_\alpha^\gamma(\vec{x}) \\ &\quad - \beta_\beta M_K(r) z_\alpha^\beta(\vec{x}). \end{aligned} \quad (\text{A.13})$$

First, the constraints eq. (A.9) and (A.10) must be valid for all times. Therefore, their derivatives with respect to time

$$\begin{aligned} \dot{\Psi}_\alpha &= \int d^3r z_\beta^\alpha(\vec{x}) \left[\dot{\tilde{\Pi}}_\beta(\vec{x}) - \frac{2}{\sqrt{3}} \lambda(r) f_{8\beta\gamma} \dot{\tilde{\eta}}_\gamma(\vec{x}) \right] = 0, \\ \dot{\chi}_\alpha &= \int d^3r z_\beta^\alpha(\vec{x}) M_K(r) \dot{\tilde{\eta}}_\beta(\vec{x}) = 0 \end{aligned} \quad (\text{A.14})$$

must also vanish. Second, we substitute the equations of motion (A.13) into these relations and extract the Lagrange multipliers,

$$\begin{aligned} \alpha_\alpha &= -\frac{R_\alpha}{\sqrt{\Theta_K}} - \frac{4}{\sqrt{3}} \int d^3r \lambda(r) z_\beta^\alpha(\vec{x}) f_{8\beta\gamma} \tilde{\eta}_\gamma(\vec{x}), \\ \beta_\alpha &= -\frac{2}{\sqrt{3}} \omega_0 f_{8\alpha\beta} \alpha_\beta - \frac{\Gamma}{2\sqrt{\Theta_K}} f_{8\alpha\beta} D_{8\beta} \\ &\quad - (m_K^2 - m_\pi^2) \int d^3r z_\beta^\alpha(\vec{x}) \tilde{\eta}_\beta(\vec{x}), \end{aligned} \quad (\text{A.15})$$

with ω_0 and Γ defined in eqs. (3.7) and (3.8), respectively. As the conjugate momenta, these Lagrange multipliers obviously carry collective

$$\begin{aligned} \alpha_\alpha^{(\text{coll})} &= -\frac{R_\alpha}{\sqrt{\Theta_K}} \quad \text{and} \\ \beta_\alpha^{(\text{coll})} &= \frac{2}{\sqrt{3}} \omega_0 f_{8\alpha\beta} \frac{R_\beta}{\sqrt{\Theta_K}} - \frac{\Gamma}{2\sqrt{\Theta_K}} f_{8\alpha\beta} D_{8\beta} \end{aligned} \quad (\text{A.16})$$

as well as fluctuation pieces: $\tilde{\alpha}_\alpha = \alpha_\alpha - \alpha_\alpha^{(\text{coll})}$ and $\tilde{\beta}_\alpha = \beta_\alpha - \beta_\alpha^{(\text{coll})}$. When substituted into the Hamiltonian, eq. (A.12) the collective pieces provide the above-

mentioned additional contribution to the Yukawa interaction. In total the latter reads,

$$H_{\text{int}} = \frac{2R_\alpha}{\sqrt{3}\Theta_K} f_{8\beta\gamma} \int d^3r z_\beta^\alpha(\vec{x}) [2\lambda(r) - \omega_0 M_K(r)] \tilde{\eta}_\gamma(\vec{x}) - \frac{4}{3} \sqrt{\Theta_K} D_{8\alpha} f_{8\alpha\beta} \int d^3r z_\gamma^\beta(\vec{x}) \times \left[m_K^2 - m_\pi^2 - \frac{3\Gamma}{8\Theta_K} M_K(r) \right] \tilde{\eta}_\gamma(\vec{x}). \quad (\text{A.17})$$

The formula, eq. (7.1) displayed in the main part of the paper finally results from replacing the zero-mode wave function according to eq. (A.1) and the intrinsic fluctuations according to eq. (5.4).

We abstain from presenting further details on the homogeneous parts of the equations of motion for $\tilde{\Pi}_\alpha$ and $\tilde{\eta}_\alpha$. It completely suffices to note that by i) projecting on the P -wave channel, ii) omitting the explicitly inhomogeneous pieces proportional to R_α and $D_{8\alpha}$ (flavor symmetry breaking) and iii) substituting the fluctuation pieces for the Lagrange multipliers: $\alpha_\alpha \rightarrow \tilde{\alpha}_\alpha$ and $\beta_\alpha \rightarrow \tilde{\beta}_\alpha$, eqs. (A.13) directly transform into eqs. (3.13) that we employed to compute the background phase shift in sect. 3. In the main text the solutions of these homogeneous parts are denoted $\bar{\Pi}_\alpha$ and $\bar{\eta}_\alpha$, respectively.

Finally we would like to comment on the treatment of the angular velocity Ω_8 and its N_C scaling. To gain some insight, we compute the time derivative of R_α from $[R_\alpha, H]$ with H given in eq. (4.5),

$$\frac{dR_\alpha}{dt} = \left(\frac{1}{\Theta_\pi} - \frac{1}{\Theta_K} \right) f_{i\alpha\beta} R_i R_\beta - \left(\frac{R_8}{\Theta_K} + \Omega_8 \right) f_{8\alpha\beta} R_\beta - \frac{\Gamma}{2} f_{8\alpha\beta} D_{8\beta}. \quad (\text{A.18})$$

While the first term, which is subject to ordering ambiguities, is suppressed by $\mathcal{O}(1/\sqrt{N_C})$, the second term, together with the constraint $R_8 = \frac{N_C}{2\sqrt{3}}$, suggests that Ω_8 should be counted as $\mathcal{O}(N_C^0)$ and thus not be omitted as $N_C \rightarrow \infty$.

By isolating the Ω_8 contribution from the time derivative of the *ansatz* eq. (2.11) we find to linear order in the fluctuations,

$$\frac{dU(\vec{x}, t)}{dt} = \frac{i}{f_\pi} \left(\frac{2}{\sqrt{3}} \omega + \Omega_8 \right) A_3(t) \sqrt{U_0(\vec{x})} \times [f_{8\alpha\beta} \lambda_\alpha \tilde{\eta}_\beta(\vec{x}, t)] \sqrt{U_0(\vec{x})} A_3(t)^\dagger + \mathcal{O}(\tilde{\eta}^2), \quad (\text{A.19})$$

where we have used $[\lambda_8, U_0(\vec{x})] = 0$. Hence Ω_8 may be absorbed in a suitable redefinition of the energy scale $\omega \rightarrow \omega' = \omega + \frac{\sqrt{3}}{2} \Omega_8$ and thus be omitted from the rotation-vibration coupling¹¹ as in eq. (5.2). However, it may not be omitted from the ‘‘classical’’ level, eq (4.1), which does not contain the fluctuation energy to compensate for Ω_8 .

¹¹ Some care is required with this argument as it only applies to local pieces in the action. For the non-local Wess-Zumino term explicit computation also leads to the linear combination, eq. (5.2).

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